K J Somaiya College of Science \& Commerce Autonomous (Affiliated to University of Mumbai)

# K. J. Somaiya College of Science and Commerce <br> Autonomous- Affiliated to University of Mumbai 

(Re-accredited "A" Grade by NAAC)

## Department of Mathematics

T. Y. B. Sc.

## Proposed Syllabus

(With effect from 2023-24)

## T.Y. B. Sc. (Mathematics) SEMESTER V <br> Core Course-I <br> COURSE TITLE: Multivariate Differential Calculus COURSE CODE: 23US5MTCC1MDC CREDITS - 02

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Analyze limits and continuity of scalar and vector functions
CLO 2: Analyze the concept of differentiability of scalar valued functions
CLO 3: Apply the concept of differentiability of scalar and vector valued functions

Module 1 Limits and continuity of functions in the $n$-dimensional Euclidean space

## Learning Objectives:

The learner should be able to:

1. Relate one coordinate system to another
2. Sketch level curves and level surfaces
3. Establish results on limits and continuity of scalar and vector valued functions
4. Solve problems on limits and continuity of scalar and vector valued functions

## Learning Outcomes:

After the successful completion of the module, the learner will be able to:

1. Solve problems of Vector Algebra
2. Apply the ideas of polar coordinates to solve problems on limits and continuity
3. Understand shapes of surfaces in terms of level curves
4. Evaluate limit and iterated limits
5. Use the tool to check the continuity of scalar and vector valued functions

| 1.1 | Vectors in n-dimensional Euclidean space. Scalar product and norm of a vector. <br> Angle between two vectors. Projection of a vector in the direction of another <br> vector. Cross product and scalar triple product in case of vectors in the three- <br> dimensional space. Geometrical interpretation | [1L] |
| :--- | :--- | :--- |
| 1.2 | Lines and planes in $\boldsymbol{R}^{\boldsymbol{n}}(\boldsymbol{n}=\mathbf{2 , 3})$. Angle between two lines and angle between <br> two planes. Angle between a line and a plane. Distance between two parallel <br> lines and distance between two parallel planes. Standard equations of quadric <br> surfaces in space: sphere, ellipsoid, paraboloid, hyperboloid, cone, cylinder. <br> Polar coordinates in the plane. Cylindrical and spherical coordinates in space. <br> Function of two variables taking real values. Its graph is a surface in space. | [4L] |


|  | Level curves. Functions of three variables and level surfaces. |  |  |
| :--- | :--- | :--- | :---: |
| 1.3 | Neighborhood of a point in $\boldsymbol{R}^{\boldsymbol{n}}$. Scalar valued functions of two or more <br> variables. limit of a function at a point in $\boldsymbol{R}^{\boldsymbol{n}}$ in terms of epsilon and delta and <br> ilterated limits. <br> Use of polar coordinates and Two path test in the evaluation of limits. <br> Continuity of scalar valued functions. Algebra of continuous functions. | [4L] |  |
| 1.4 | Vector valued functions. Limits and continuity of vector valued functions. | [3L] |  |
|  |  |  |  |

- T. M. Apostol; Calculus (Vol. I); John Wiley and Sons (Asia) P. Ltd.
- T. M. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- James Stewart; Multivariable Calculus


## Additional Reference books:

- G.B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1992
- D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989
- Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis (12th Edition, 1979), S. Chand and Co
- M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995


## Module 2

## Learning Objectives

The learner should be able to:

1. Analyse the concepts of Differentiability :
2. Compute directional derivative, partial derivatives, gradient and Jacobian matrix

## Learning outcomes

The learner will be able to:

1. Use the definition to solve problems
2. Apply tools developed to determine directional derivative gradient, partial derivatives and Jacobian matrix
3. Apply the chain rule

| 2.1 | Derivative of a scalar valued function with respect to a vector, directional <br> derivatives and partial derivatives. <br> Definition of differentiability of a scalar valued function of several variables at | [4L] |
| :--- | :--- | :--- |


|  | a point in terms of a linear transformation. Uniqueness of the derivative as a <br> linear transformation. Sum and product rule, Derivative of a scalar multiple of <br> a differentiable function. Differentiability of scalar valued function implies <br> continuity |  |
| :--- | :--- | :--- |
| $\mathbf{2 . 2}$ | Continuity of partial derivatives and relationship between Derivative of a <br> scalar valued function and its gradient. Chain rule for scalar valued functions <br> Clairaut's Theorem. Euler's theorem for homogeneous functions. | [4L] |
| $\mathbf{2 . 3}$ | Differentiation of vector valued functions in terms of linear transformation <br> and related results <br> Jacobian matrix of a differentiable function. <br> Chain rule of differentiation and its verification using Jacobian matrices. | [4L] |
| Reference books |  |  |

- T. M. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- James Stewart; Multivariable Calculus
- Jerrold E. Marsden, Anthony Tromba Vector Calculus; Basic multivariable calculus


## Additional Reference books:

- G.B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1992
- D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989
- Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis (12th Edition, 1979), S. Chand and Co
- M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995


## Module 3 Mean value theorem, Taylor's theorem and applications

## Learning Objectives

The learner should be able to

1. Prove Mean value theorem, Taylor's Theorem and other results.
2. Apply results to solve real life problems

## Learning outcomes

The learner will be able to:

1. Comprehend the geometric interpretation of gradient
2. Verify Mean Value Theorem
3. Apply Taylor's Theorem to solve problems on approximations and extreme values

| 4. Use Lagrange's Multiplier to find Extreme Values |  |  |
| :--- | :--- | :--- |
| $\mathbf{3 . 1}$ | Mean value theorem in case of scalar valued functions of several variables. Its <br> illustration for two or three variables. | [2L] |
| $\mathbf{3 . 2}$ | Linearization of a scalar valued function in the neighborhood of a point. <br> Computation of tangent planes and normal vectors to a surface using gradient. <br> Related results. | [3L] |
| $\mathbf{3 . 3}$ | Taylor's formula of first order and second order in case of scalar valued <br> functions. Computation of approximate values using this formula. | [3L] |
| $\mathbf{3 . 4}$ | Critical points, Hessian matrix, Computation of Extreme values of a scalar <br> valued function and saddle points using second derivative test. <br> Use of Lagrange multipliers to determine extreme values. | [4L] |

## Reference books

- T. M. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- James Stewart; Multivariable Calculus
- Jerrold E. Marsden, Anthony Tromba Vector Calculus; Basic multivariable calculus


## Additional Reference books:

- G.B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1992
- D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989
- Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis (12th Edition, 1979), S. Chand and Co
- M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995

> T.Y. B. Sc. (Mathematics) SEMESTER V
> Core Course- II
> COURSE TITLE: Algebra-II
> COURSE CODE: 23US5MTCC2ALG CREDITS - 02

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Apply Caley Hamilton theorem and Diagonalization results.
CLO 2: Identify quadrics and conics using orthogonal change of variables.
CLO 3: Apply various properties of groups to solve related problems.

## Learning Objectives:

The learner should be able to:

1. Understand eigen values, eigen vectors and related results
2. Prove Cayley Hamilton theorem; necessary \& sufficient condition for diagonalizability.
3. Apply Cayley Hamilton theorem.
4. Apply the results proved to diagonalize a matrix.

## Learning Outcomes:

After the successful completion of the module, the learner will be able to:

1. Solve problems related to eigen values, eigen vectors
2. Prove results such as Cayley Hamilton theorem; Diagonalization criteria
3. Apply Cayley Hamilton theorem
4. Use the tools developed to diagonalize a matrix and linear transformation.

| 1.1 | Eigen values and eigen vectors, characteristic polynomial, eigen space, <br> algebraic and geometric multiplicity, related results. | [4L] |
| :--- | :--- | :--- |
| 1.2 | Cayley Hamilton theorem and its applications. | [2L] |


| 1.3 | Simila and s geom | matrices, diagonaliz ficient conditions fo ric multiplicity. |  | [6L] |
| :---: | :---: | :---: | :---: | :---: |
|  | erence <br> - Lin <br> - Lin <br> - Line <br> itional S. L L. S <br> - TB | oks <br> Algebra : A geome <br> Algebra- Hoffmann <br> algebra - Gilbert S <br> ference books: <br> g- Introduction to L <br> th- Linear Algebra, <br> choff and J Wermer |  |  |
|  | dule 2 | Quadratic Forms |  |  |
| Learning Objectives |  |  |  |  |
| The learner should be able to: |  |  |  |  |
| 1. Apply the concepts of orthogonal diagonalization. |  |  |  |  |
| 2. Prove the results related to orthogonal diagonalization and quadratic form |  |  |  |  |
| 3. Reduce a quadratic form |  |  |  |  |
| 4. Compute index, rank and signature of a quad |  |  |  |  |
| 5. Determine the definiteness of a quadratic form |  |  |  |  |

## Learning outcomes

The learner will be able to:

1. Find an orthogonal change of variables to reduce a quadratic form in standard form.
2. Apply tools developed to identify the conic or quadric represented by quadratic form.
3. find definiteness of given quadratic form
4. Compute index, rank and signature of a quadratic form

| 2.1 | Orthogonally diagonalizable matrix, a real symmetric matrix has real eigen <br> values. Eigen vectors of a real symmetric matrix with respect to distinct eigen <br> values are orthogonal. | [5L] |
| :--- | :--- | :--- |
| 2.2 | Quadratic form in n variables, symmetric matrix associated with a quadratic <br> form, reducing a quadratic form to a sum of squares (standard form), index, <br> rank and signature of a quadratic form. | [4L] |

2.3 Identification of a conic and quadric surface using quadratic form, positive definite, negative definite and indefinite forms.

## Reference books

- Linear Algebra : A geometric approach- S. Kumareson
- Linear Algebra- Hoffmann and Kunze
- Linear algebra - Gilbert Strang

Additional Reference books:

- S. Lang- Introduction to Linear Algebra
- L. Smith- Linear Algebra, Springer.
- T Banchoff and J Wermer- Linear Algebra, Prentice Hall of India.


## Module 3

## Learning Objectives

The learner should be able to

1. Prove various results in a group
2. Study cyclic group and its properties
3. Appreciate groups such as Dihedral group, Quaternion group
4. Identify group homomorphism and find its kernel

## Learning outcomes

The learner will be able to:

1. Prove results related to concepts in a group.
2. Find number of generators, number of subgroups of a cyclic group
3. Generators of a dihedral group of order six and eight
4. Solving problems in a group and subgroup
5. Verify properties of group homomorphism

| 3.1 | Review of groups and subgroups; properties and examples. | [1L] |
| :--- | :--- | :--- |
| 3.2 | Order of a group, order of an element in a group and the properties, center <br> and normalizer. | [2L] |
| 3.3 | Permutation groups, groups of rotations and reflection of a regular polygon, <br> the alternating group An, related results. | [1L] |


| 3.4 | Cyclic group, definition and examples, subgroups of a cyclic group is cyclic, number of generators of a cyclic group, describing all subgroups of a cyclic group, groups generated by two elements | [4L] |
| :---: | :---: | :---: |
| 3.5 | Cosets, Langrange's theorem | [2L] |
| 3.6 | Homomorphisms and isomorphism of groups, examples and properties, kernel of a homomorphism. | [2L] |
|  | rence books: <br> I.N.Herstein - Topics in Algebra <br> J. Gallion - contemporary abstract algebra <br> J.B. Fraliegh- A first course in abstract algebra <br> M. Artin - Algebra <br> N.S. Gopalkrishnan - University Algebra <br> Additional Reference books: <br> D. Dummit and R. Foote- Abstract algebra Bhattacharya Jain Nagpal - Abstract Algebra |  |

T.Y. B. Sc. (Mathematics) SEMESTER V<br>Core Course- III<br>COURSE TITLE: Topology of Metric spaces - I COURSE CODE: 23US5MTCC3TMS1 CREDITS - 02]

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:

1. Describe different examples of metrics over $R, C$ and other spaces
2. Apply properties of sequence in a metric to solve problems
3. Understand consequences of completeness

| Module 1 | Introduction to Metric space | [12L] |
| :--- | :--- | :--- |

## Learning Objectives:

1. Study of different types of metrics
2. Comprehend open sets, closed sets and limit points, interior points, exterior points and boundary points.

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Prove results related to metric space
2. Solve problems on metrics
3. Find special points of a set with respect to a given metric

| 1.1 | Definition of Metric space, examples of metric spaces $\mathbb{R}, \mathbb{R}^{2}, \mathbb{C}$, |
| :--- | :--- |

Discrete metric space.
Normed linear space, Metric induced by the norm; Euclidean space $\mathbb{R}^{n}$ with Euclidean, sup and sum metric; $\boldsymbol{\ell}^{1}$ and $\boldsymbol{l}^{2}$ spaces of sequences. Translation invariance of the metric induced by the norm.
The space of all bounded real valued functions defined on an interval [ $\mathrm{a}, \mathrm{b}$ ] and the metric induced by sup norm. The space of real valued continuous functions on [a,b].
Metric subspaces, Product of two metric spaces.

| 1.2 | Open ball and open set in a metric space, examples of open sets in various metric spaces, Hausdorff property <br> Definition of closed set as a complement of an open set. <br> Results based on taking union or intersection of one or more open sets and closed sets. <br> Limit point of a set. Characterization of closed set as a set which contains all its limit points. <br> Interior and closure of a set. Boundary of a set. Properties of open sets, Structure of an open set in $\mathbb{R}$. | [6L] |
| :---: | :---: | :---: |
| Reference books <br> - Metric Spaces P. K. Jain and Khalid Ahmad, Narosa Publications <br> - Copson, E. T. (1996) Metric Spaces. Universal Book Stall, New Delhi. Additional Reference books: <br> - Kumaresan, S. (2005) Topology of Metric spaces |  |  |
| Mod | 2 Sequence in a metric space | [12L] |
| Learning Objectives <br> The aim of the module is to: <br> 1. Study equivalence of metrics <br> 2. Learn convergent sequences in a metric space <br> 3. Characterize limit points |  |  |
| Learning Outcomes <br> The learner will be able to: <br> 1. Apply properties of convergent sequences <br> 2. Understand dense sets and separability |  |  |
| 2.1 | Equivalent metrics. Distance of a point from a set, distance between two sets, diameter of a set in a metric space, bounded sets. | [3L] |
| 2.2 | Sequence in a metric space, Bounded sequence, Convergent sequence, Uniqueness of limit, Cauchy sequence. Every convergent sequence is Cauchy but the converse is false. Subsequence of a sequence, examples and results related to convergent and Cauchy sequence. | [5L] |
| 2.3 | Characterization of limit points and closure points in terms of sequences. Definition and examples of open and closed sets in subspaces, Dense subsets in a metric space and Separability. | [4L] |

## Reference books

- Metric Spaces P. K. Jain and Khalid Ahmad, Narosa Publication
- Copson, E. T. (1996) Metric Spaces. Universal Book Stall, New Delhi Additional Reference books:
- Kumaresan, S. (2005) Topology of Metric spaces

| Module 3 | Complete metric spaces. | $[12 \mathrm{~L}]$ |
| :--- | :--- | :--- |


| Learning Objectives |
| :--- |
| The aim of the module is to: |
| 1. Study complete metric space |
| 2. Characterize completeness |
| 3. Understand contraction principle |


| Learning Outcomes |
| :--- |
| The learner will be able to: |
| 1. Decide completeness of a metric space <br> 2. Verify a set with respect to denseness <br> 3. Apply results proved |
| 3.1 |
| Definition of complete metric space, Examples of complete metric <br> spaces. <br> A subset of a complete metric space is complete metric space $X$ if and <br> only if it is closed in $X$. <br> Interpretation of uniform convergence as convergence in the complete <br> metric space B[a,b] under the metric given by \|| ||sup. C[a,b], the set <br> of all continuous real valued functions defined on an interval [a,b], is <br> a closed subset of B[a,b] under the metric induced by \|| || sup. |
| [3L] |
| 3.2 |
| Review of Nested Intervals property in $\mathbb{R}$. <br> Generalization of Nested Intervals Property to Cantor's Intersection <br> Property in a complete metric space. <br> Characterization of complete metric spaces using Cantor Intersection <br> Theorem. |
| [3L] |


| $\mathbf{3 . 3}$ | Applications of Cantor Intersection Theorem. Density of rational <br> numbers. Decimal representation of real numbers. Bolzano- <br> weierstrass theorem for $\mathbb{R}$ under usual metric. Intermediate value <br> property of real valued continuous functions defined over closed and <br> bounded intervals. | [3L] |
| :--- | :--- | :--- |
| 3.4 | Completion of a metric space. Every metric space can be completed. <br> Contraction Mapping and Banach's fixed point theorem. | [3L] |
| Reference books <br> 1. Metric Spaces P. K. Jain and Khalid Ahmad, Narosa Publications <br> 2. Copson, E. T. (1996) Metric Spaces. Universal Book Stall, New Delhi. <br> Additional Reference books: <br> 1. Kumaresan, S. (2005) Topology of Metric spaces |  |  |

## T.Y. B. Sc. (Mathematics) SEMESTER V <br> Core Course- IV <br> COURSE TITLE: -Real Analysis III <br> COURSE CODE: 23US5MTCC4RA CREDITS - 02]

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1. Apply results proved on sequence of real valued functions
CLO 2. Discuss convergence and uniform convergence of series of real valued functions
CLO 3. Express a real valued function as a series in terms of $\sin$ and cos functions

| Module 1 | Sequence of functions |
| :--- | :--- |

[12L]

## Learning Objectives:

This module is intended to:

1. Study point wise and uniform convergence of sequences of real valued functions
2. Solving problems on uniform convergence

Learning Outcome:
After the successful completion of the module, the learner will be able to:

1. Prove results associated with uniform convergence of sequences of real valued functions
2. Solve problems on pointwise and uniform convergence of sequences of real valued functions

| 1.1 | Pointwise convergence of sequence of functions with examples <br> Uniform convergence of sequence of functions with examples <br> Uniform convergence implies pointwise convergence, examples to show <br> converse is not true. | [6L] |
| :--- | :--- | :--- |
| 1.2 | Consequences of uniform convergence of sequences of functions on <br> Continuity with examples <br> Consequences of uniform convergence of sequences of functions on <br> Integrability with examples <br> Consequences of uniform convergence of sequences of functions on <br> differentiability with examples <br> Use of second derivative test to decide the maximum difference between <br> function and the limit function at any point in the domain. <br> Cauchy's criterion for uniform convergence and its equivalence with the <br> definition. Mnt test for convergence of sequence of functions. | [6L] |

## Reference books

1. Richard R. Goldberg ,Methods of Real Analysis, Oxford \& IBH Publishing Co Pvt.Ltd
2. Robert G. Bartle, Donald R. Sherbert ,Introduction to Real Analysis, John Wiley and sons 2000
3. Apostol, Mathematical Analysis, Narosa Publishing house Additional Reference books:
4. Principles of Mathematical Analysis, Walter Rudin,third edition, McGraw-Hill, Inc.

Module 2 Series of functions
[12L]

Learning Objectives:
This module is intended to:

1. Study point wise and uniform convergence of series of real valued functions
2. Solving problems on pointwise and uniform convergence of series of real valued functions

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Solve problems on uniform convergence
2. 

Compute radius and interval of convergence

| 2.1 | Point wise convergence and Uniform convergence of series of functions | [6L] |
| :--- | :--- | :--- | Weierstrass' M-test. Examples Consequences of uniform convergence of series of functions on continuity Consequences of uniform convergence of series of functions on Integrability, examples Consequences of uniform convergence of series of functions on differentiability, examples

Term by term differentiation and integration, simple problems based on it.
2.2 Definitions of radius of convergence and interval of convergence of a real

Justification of above concepts and techniques with r.t. Uniform convergence.

Reference books

1. Richard R. Goldberg ,Methods of Real Analysis, Oxford \& IBH Publishing Co Pvt.Ltd
2. Robert G. Bartle, Donald R. Sherbert ,Introduction to Real Analysis, John Wiley and sons 2000
3. Apostol, Mathematical Analysis, Narosa Publishing house

Additional Reference books:

1. Principles of Mathematical Analysis, Walter Rudin,third edition, McGraw-Hill, Inc.

| Module 3 | Fourier Series | [12L] |
| :--- | :--- | :--- |
| Learning Objectives: <br> This module is intended to: <br> 1. Study Fourier series on C[- $\pi, \pi]$ <br> 2. |  |  |
| Learning Outcome: <br> After the successful completion of the module, the learner will be able to: <br> 1. Find Fourier series of a real valued function <br> 2. Apply the results proved. | [4L] |  |
| 3.1 | Fourier series of functions on C[- $\pi, \pi]$ | [4L] |
| 3.2 | Dirichlet kernel, Fejer kernel, Cesaro summability of Fourier series of <br> functions on C[- $\pi, \pi]$ | [4L] |
| 3.3 | Bessel's inequality and Pareseval's identity, <br> Convergence of the Fourier series in $l^{2}$ norm, Fourier series is best <br> approximation in terms of trigonometric functions of a function in $l^{2}$ |  |
| Reference books |  |  |
| 1. Richard R. Goldberg ,Methods of Real Analysis, Oxford \& IBH Publishing Co Pvt.Ltd |  |  |
| Additional Reference books: |  |  |
| 1. Apostol, Mathematical Analysis, Narosa Publishing house |  |  |

## T. Y. B. Sc. (Mathematics)

SEMESTER V - Practical COURSE CODE: 23USMTCCP1 Credit- 02

## Learning Objectives:

The Practical is intended to

1. Solve problems based on the concepts learnt
2. Apply the concepts in various situation

## Learning Outcome:

After the successful completion of the practical, the learner will be able to:

1. Solve problems
2. Apply the results proved
3. Generate examples and counterexamples

## Module I Multivariate Differential Calculus

1.1 Problems on dot product, cross product, box product, calculating length of a vector, angle between lines, area of parallelograms, volume of parallelepiped. Finding equations of lines and planes in space. Identifying standard quadric surfaces. Conversion between cartesian coordinates and polar, cylindrical and spherical coordinates. Sketching of simple curves in polar coordinates. Sketching level curves.
1.2 Problems on iterated limits, limits and continuity in case of scalar valued functions. Use of two path tests and use of polar coordinates. Problems on limits and continuity in case of vector valued functions
1.3 Computation of partial derivatives and directional derivatives. Use of gradients in computing the directional derivatives. Computing the derivative of a homogeneous function.
1.4 Computation of Jacobian matrices. Verification of chain rule. Verification of Mean value theorem in case of scalar valued functions.
1.5 Computing linearization of a scalar valued function of two or more variables. Tangential planes and normal vectors to surfaces at a given point
1.6 Use of Taylor's second order formula in finding approximate values of complicated functions. Finding extrema and saddle points of a scalar valued function of two variables. Computing extreme values using Lagrange's multipliers.

\section*{| Module 2 | Algebra II |
| :--- | :--- |}

2.1 Problems based on Eigen values and eigen vectors
2.2 Problems based on Cayley Hamilton theorem
2.3 Problems based on Diagonalization of a matrix
2.4 Problems based on Quadratic forms
2.5 Problems based on Groups and Subgroups
2.6 Problems based on Group Homomorphisms

## T. Y. B. Sc. (Mathematics)

## SEMESTER V - Practical <br> COURSE CODE: 23USMT5CCP2 Credit- 02

## Learning Objectives:

The Practical is intended to

1. Solve problems based on the concepts learnt
2. Apply the concepts in various situation

## Learning Outcome:

After the successful completion of the practical, the learner will be able to:

1. Solve problems
2. Apply the results proved
3. Generate examples and counterexamples

## Module I Topology of Metric spaces - I

1.1 Problems on various examples of metric spaces
1.2 Problems on Open sets, closed sets, interior points, limit points, boundary of a set. Drawing open balls in a given metric space.
1.3 Problems on equivalent metric, distance between two sets, diameter of a set.
1.4 Problems on convergent sequences and cauchy sequences, Limit point as a limit of a sequence. characterization of closed sets in terms of limit points.
1.5 Problems on complete metric spaces and Cantor intersection theorem
1.6 Problems on Applications of Cantor intersection theorem, contraction mapping and fixed point theorem.

| Module II | Real Analysis III |
| :--- | :--- |

2.1 Problems on Pointwise convergence of sequence of functions and Uniform convergence of sequence of functions
2.2 Problems on Consequences of uniform convergence of sequences of functions on Continuity, Integrability, differentiability. Use of second derivative test to decide the maximum difference between function and the limit function at any point in the domain. Problems on Cauchy's criterion for uniform convergence, Mn test for convergence of sequence of functions.
2.3 Problems on Point wise convergence and Uniform convergence of series of functions, Weierstrass' M-test, Consequences of uniform. convergence of series of functions on continuity, Integrability, differentiability.
2.4 Problems on Term by term differentiation and integration, radius of convergence and interval of convergence of a real power series,
2.5 Problems on Fourier series of functions on $\mathrm{C}[-\pi, \pi]$, Dirichlet kernel, Fejer kernel, Cesaro summability of Fourier series of functions on $\mathrm{C}[-\pi, \pi]$
2.6 Problems on Bessel's inequality and Pareseval's identity, Convergence of the Fourier series in 12 norm.

## TYBSC ( MATHEMATICS) SEMESTER VI

Discipline Specific Elective Courses
Students are required to select any two courses

Of the following DSE courses, learners can select any two of the courses. The courses having maximum number of votes will be considered for the year and subject to availability of the concerned teacher. A learner can also opt for Swayam/NPTEL course as one of the DSE courses after it is vetted by the department.

| DSE | Title |
| :--- | :--- |
| DSE 1 | Number Theory and its applications I |
| DSE 2 | Algorithm and C Programming |
| DSE 3 | Advanced Graph Theory |
| DSE 4 | Financial Mathematics |

## T.Y. B. Sc. (Mathematics) SEMESTER V <br> Discipline Specific Course-I <br> COURSE TITLE: Number Theory and its applications I COURSE CODE: 23US5MTDS1NTA CREDITS - 02]

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Prove standard theorems in Number Theory
CLO 2: Find solutions of Diophantine equations
CLO 3: Find solutions of quadratic congruences

\section*{| Module 1 | Prime numbers and congruences |
| :--- | :--- | [12L]}

## Learning Objectives

The learner should be able to:

1. Prove the results involving congruence
2. Apply results proved in solving problems

## Learning Outcomes

After the successful completion of the module, the learner will be able to:

1. Prove the results involving congruence
2. Apply theorems proved to solve problems related to congruence

| 1.1 | Review: Divisibility, Definition of Prime, Definition and elementary properties of Congruence, The Fundamental Theorem of Arithmetic, Number of primes are infinite, Distribution of Primes (There are arbitrarily large gaps between consecutive primes). (No questions from this in the semester end exam). | [2L] |
| :---: | :---: | :---: |
| 1.2 | Definition and examples of Twin primes, linear congruence, Complete Residue system modulo m, Reduced residue system modulo m, Euler's Phi Function, Euler's Phi function is multiplicative. | [4L] |
| 1.3 | Fermat little Theorem, Euler's generalization of Fermat Little Theorem, Wilson Theorem, Chinese Remainder theorem. | [6 |
| Reference books <br> 1. David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw Hill <br> 2. H. Zuckerman and H. Montgomery. Elementary number theory. John Wiley \& Sons. Inc. <br> Additional Reference books <br> 1. G. H. Hardy, and E.M. Wright, An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981 |  |  |
| Module 2 Diophantine equations and their solutions |  | [12L] |
| Learning Objectives <br> The learner should be able to: <br> 1. Study solutions of Diophantine equations <br> 2. Solve problems based on proved results |  |  |
| Learning outcomes <br> The learner will be able to: <br> 1. Solve linear Diophantine equation. <br> 2. Find solutions of Pythagorean triples <br> 3. Prove results associated with higher order Diophantine equations |  |  |
| 2.1 | The linear equations $a x+b y=c$. Representation of Prime as sum of two squares, Thue's Lemma, Representation of integers of the type $N^{2} m$ for square free $m$ as sum of two squares. | [6L] |
| 2.2 | Pythagorean triples and their solutions. $x^{4}+y^{4}=z^{2}$ has no solutions in positive integers, $x^{4}-y^{4}=z^{2}$ has no solutions in positive integers. | [4L] |
| 2.3 | The radius of the inscribed circle of a Pythagorean triangle is integer. The area of a Pythagorean triangle can never be a perfect square. | [2L] |
| Reference books <br> 1. David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw Hill <br> 2. H. Zuckerman and H. Montgomery. Elementary number theory. John Wiley \& Sons. Inc. |  |  |

## Additional Reference books

1. G. H. Hardy, and E.M. Wright, An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981

## Module 3 Quadratic Reciprocity <br> [12L]

## Learning Objectives

The learner should be able to:

1. Understand the Legendre symbol and Jacobi symbol
2. Solve problems based on proved results

## Learning outcomes

The learner will be able to:

1. Prove results associated to Quadratic Reciprocity
2. Compute Legendre and Jacobi symbol
3. Decide about solvability of a quadratic congruence with prime modulus as well as composite modulus

| 3.1 | Quadratic Residues, Euler's Criterion with proof. Legendre symbol and its properties. | [4L] |
| :--- | :--- | :--- |
| 3.2 | Gauss Lemma, Quadratic Reciprocity Law. | [4L] |
| 3.3 | There are infinitely many primes of the form 4k+1, 8k-1, 6k+1. For a prime $p$ <br> characterizing (2/p), (3/p), (5/p). Solving quadratic congruence equations. <br> The Jacobi Symbol and law of Reciprocity for Jacobi Symbol. | [4L] |
| Reference books |  |  |

1. David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw Hill
2. H. Zuckerman and H. Montgomery. Elementary number theory. John Wiley \& Sons. Inc.

Additional Reference books

1. G. H. Hardy, and E.M. Wright, An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981

## T.Y. B. Sc. (Mathematics) SEMESTER V <br> Discipline Specific Course- II <br> COURSE TITLE: Algorithm and C Programming COURSE CODE: 23US5MTDS2ACP CREDITS - 02]

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Write algorithm for solving mathematical problems using syntaxes of C programming

CLO 2: Implement various mathematical concepts using C
CLO 3: Write User defined functions for various mathematical concepts

## Module 1 Algorithm and Introduction to C Programming

## Learning Objectives:

The learner should be able to:

1. Comprehend logic development
2. Understand the syntax used in C
3. Use various data type, Input/output statements in C

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Develop program logic for a given problem
2. Express C concepts with examples
3. Use different data type and qualifiers and operators as per the requirement of a problem
4. Typecast as per requirement

| 1.1 | Algorithm <br> Fundamentals of algorithm <br> Counting, exchange, <br> Summation of a set of numbers, factorization, prime factorization, <br> checking primes, sorting, searching, matrix multiplication, maximum, <br> minimum in an array | [5L] |
| :--- | :--- | :--- |
| 1.2 | Structure of C program: Header and body, Concept of header files, Use of <br> comments, Compilation of a program. <br> Storage classes in C. | [1L] |
| 1.3 | Data Concepts: Variables, Constants, data types like: int, float char, <br> double and void. Qualifiers: short and long size qualifiers. Declaring <br> variables, Scope of the variables according to block, Hierarchy of data <br> types. | [2L] |


| 1.4 | Types of operators: Arithmetic, Relational, Logical, Compound <br> Assignment, Increment and decrement, Conditional or ternary operators. <br> Precedence and order of evaluation. Statements and Expressions. | [2L] |
| :--- | :--- | :--- |
| 1.5 | Type conversions: Automatic and Explicit type conversion. <br> Data Input and Output functions: Formatted I/O: printf(), scanf(). <br> Character I/O : getch(), gets(), putch(), puts(). <br> Numeric Mathematical functions such as pow(), sqrt(), etc. | [2L] |

## Reference book

- Let us C by Yashwant Kanetkar, BPB


## Additional References:

- Programming in ANSI C (Third Edition) : E Balagurusamy, TMH
- Programming with C by Byron Gottfried, Schaum's outlines, Mc Graw Hill.
- Mastering Algorithms with C, Kyle Loudon, Shroff Publishers.
- Algorithms in C (Third Edition): Robert Sedgewick, Pearson Education Asia.
- Programming in ANSI C by Ram Kumar, Rakesh Agrawal, TMH.
- Programming with C (Second Edition): Byron S Gottfried (Adapted by Jitender Kumar Chhabra) Schaum's Outlines (TMH)
- Programming with C : K R Venugopal, Sudeep R Prasad TMH Outline Series.


## Module 2 Control statements and arrays

[12L]
Learning Objectives
The learner should be able to:

1. Comprehend the use of arrays
2. Use conditional statements, various loops and loop interrupters

## Learning outcomes

The learner will be able to:

1. understand the syntax and use of various branching statements and loops
2. Apply control transfer statements
3. Apply break and continue statements as per requirement
4. Apply arrays to deal with different scenarios
5. Analyze the program for errors and predict the output

| 2.1 | Control statements for decision making: Branching: if statement, if-else <br> statement, if-else-if statement <br> Switch statement | [3L] |
| :--- | :--- | :--- |
| 2.2 | Looping: while loop, do- while, for loop, nested loop. <br> Loop interruption statements: break, continue. | [4L] |
| 2.3 | Arrays: One- and two- dimensional arrays. <br> declaring array variables, Array initialization, bound checking, accessing <br> array elements. | [5L] |

## Reference book

- Let us C by Yashwant Kanetkar, BPB


## Additional References:

- Programming in ANSI C (Third Edition) : E Balagurusamy, TMH
- Programming with C by Byron Gottfried, Schaum's outlines, Mc Graw Hill.
- Mastering Algorithms with C, Kyle Loudon, Shroff Publishers.
- Algorithms in C (Third Edition): Robert Sedgewick, Pearson Education Asia.
- Programming in ANSI C by Ram Kumar, Rakesh Agrawal, TMH.
- Programming with C (Second Edition): Byron S Gottfried (Adapted by Jitender Kumar Chhabra) Schaum's Outlines (TMH)
- Programming with C : K R Venugopal, Sudeep R Prasad TMH Outline Series.

```
\begin{tabular}{l|l} 
Module 3 & Functions in C
\end{tabular} [12L]
```


## Learning Objectives

The learner should be able to:

1. Understand the syntax and use of various string functions
2. Implement user defined simple and recursive functions.
3. Apply macros

## Learning outcomes

The learner will be able to:

1. Introduce String functions and
2. Implement user defined functions
3. Apply the principle of recurring function
4. Apply macros such as \#define directives etc

| 3.1 | Strings, String Functions: strlen(), strcpy(), strcat(), strcmp(). <br> Array of characters | [3L] |
| :--- | :--- | :--- |
| 3.2 | The C Preprocessor: <br> Macro expansion: Macros with arguments, <br> \#define directives, \#if and \#elif directives. | [3L] |
| 3.3 | User defined Functions: Function definition, return statement, calling a <br> function. | [4L] |
| 3.4 | Recursion: Definition, Recursion functions | [2L] |

## Reference book

- Let us C by Yashwant Kanetkar, BPB


## Additional References:

- Programming in ANSI C (Third Edition) : E Balagurusamy, TMH
- Programming with C by Byron Gottfried, Schaum's outlines, Mc Graw Hill.
- Mastering Algorithms with C, Kyle Loudon, Shroff Publishers.
- Algorithms in C (Third Edition): Robert Sedgewick, Pearson Education Asia.
- Programming in ANSI C by Ram Kumar, Rakesh Agrawal, TMH.
- Programming with C (Second Edition): Byron S Gottfried (Adapted by Jitender Kumar Chhabra) Schaum's Outlines (TMH)
- Programming with C : K R Venugopal, Sudeep R Prasad TMH Outline Series.


## T.Y. B. Sc. (Mathematics) SEMESTER V <br> Discipline Specific Course- III COURSE TITLE: Advanced Graph Theory COURSE CODE: 23US5MTDS2AGT CREDITS - 02]

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Analyze results to solve problems of Graph theory
CLO 2: Analyze results on matching to solve problems related to matching
CLO 3: Apply results on Planar graph and Polya theory to solve related problems

## Module 1 Graphs

## Learning Objectives:

The learner should be able to:

1. Understand some of the basic concepts in Graph theory
2. Understand connectivity

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Apply Menger's theorem to certain graphs.
2. Construct a simple communication network.

| 1.1 | Review: Graphs, Subgraphs, Graph isomorphism, incidence and <br> adjacency matrix, vertex degree, walks, trails and paths <br> Trees <br> Cut vertex, cut edges (bridges) Vector space associated with graph, <br> Cayley's formula and the connector problem | [5L] |
| :--- | :--- | :--- |
| 1.2 | operations on graphs, degree sequences. <br> Distance, cut vertices, cut-edges, blocks, connectivity, weighted graphs. <br> Vertex and Edge connectivity-Result к $\leq 0 \leq \delta$, Block-cut point theorem, <br> Construction of reliable communication network, Menger's theorem. | [7L] |

## Reference books

- Graph theory with application by J. A. Bondy and U. S. R. Murty (Freely downloadable)
- Graph Theory by Reinhard Diestel Electronic edition Springer Verlag. (Freely downloadable)
- Graph theory with application, Narsingh DeoPrentice Hall publication


## Learning Objectives

The learner should be able to:

1. Understand some of the concepts in Matching

| 2. Apply Matching for assignment problems |  |  |  |
| :---: | :---: | :---: | :---: |
| Learning outcomes <br> The learner will be able to: <br> 1. Apply concepts of Matching in bipartite graphs <br> 2. Apply Matching to assignment problems |  |  |  |
| 2.1 | Mat | ing, Matc | [8L] |
| 2.2 | App <br> The <br> Opt | ations: ersonnel a al assignm | [4L] |
| Reference books <br> - Graph theory with application by J. A. Bondy and U. S. R. Murty (Freely downloadable) <br> - Graph Theory by Reinhard Diestel Electronic edition Springer Verlag. (Freely downloadable) <br> - Graph theory with application, Narsingh DeoPrentice Hall publication |  |  |  |
| Mod | ule 3 | Planar gr | [12L] |
| Learning Objectives <br> The learner should be able to: <br> 1. Understand graph coloring <br> 2. find Chromatic polynomial <br> 3. Apply concepts learnt in various situation. |  |  |  |
| Learning outcomes <br> The learner will be able to: <br> 1. verify planarity of a graph <br> 2. Find the minimum chromatic number <br> 3. Find chromatic polynomial <br> 4. correlate the relation of roots and chromatic number vis-à-vis chromatic polynomials. |  |  |  |
| 3.1 | Plan sim follo $\mathrm{e} \geq$ Chr grap Chr equ | graph and connecte ing inequa and $\mathrm{e} \leq 3$ <br> matic polyn wheel etc natic equiv lent. | [6L] |
| 3.2 |  | theorem cation | [6L] |


|  | Timetabling |  |
| :--- | :--- | :--- |
| 3.3 | Polya theory |  |
| Reference books |  |  |
| 1) Graph theory with application by J. A. Bondy and U. S. R. Murty |  |  |
| (Freely downloadable) |  |  |
| 2) Graph Theory by Reinhard Diestel Electronic edition Springer Verlag. (Freely |  |  |
| downloadable) |  |  |
| 3) Graph theory with application, Narsingh DeoPrentice Hall publication |  |  |

## T.Y. B. Sc. (Mathematics) SEMESTER V <br> Discipline Specific Course- III COURSE TITLE: Financial Mathematics COURSE CODE: 23US5MTDS3FIN

## Course Learning Outcomes

After the successful completion of the Course, the learner will be able to:
CLO 1: Apply principles of no arbitrage and solve problems based on forward contract and options

CLO 2: Solve problems of evaluating non risky assets
CLO 3: Manage portfolio using different models

Module 1 Introduction: A Simple Market Model

## Learning Objectives

The learner should be able to

1. Understand various models
2. Calculate returns
3. Solve problems based on forward contract, options
4. Manage risk with options

## Learning Outcomes

At the end of the module, the learner will be able to

1. Differentiate various models
2. Apply principles to solve a problems based on on different models
3. Calculate returns
4. Solve problems based on forward contract, options
5. Manage risk with options

| $\mathbf{1 . 1}$ | Basic Notions and Assumptions | [1L] |
| :--- | :--- | :--- |
| $\mathbf{1 . 2}$ | No-Arbitrage Principle | [1L] |
| $\mathbf{1 . 3}$ | One-Step Binomial Model, Application to the Binomial Tree Model | [1L] |
| $\mathbf{1 . 4}$ | Risk and Expected Return on a Portfolio | [2L] |
| $\mathbf{1 . 5}$ | Forward Contracts | [2L] |


| 1.6 | Call | and Put Options | [2L] |
| :---: | :---: | :---: | :---: |
| 1.7 | Man | aging Risk with Options | [3L] |
| References: <br> - Mathematics for finance: An Introduction to financial Engineering by Marek Capinski and Tomasz Zastawniak <br> - An Introduction to Mathematical Finance with Applications: Understanding and Building Financial Intuition; Arlie O. Petters, Xiaoying Dong; Springer <br> - Mathematics for Finance: An Introduction to Financial Engineering;by Marek Capiński, Tomasz Zastawniak; Springer |  |  |  |
| Mod |  | Risk-Free Assets | [12L] |
| Learning Objectives <br> The learner should be able to <br> 1. Study various non risky investment <br> 2. Calculate the rate of returns in these type of investment |  |  |  |
| Learning Outcomes <br> At the end of the module, the learner will be able to <br> 1. Calculate the rate of returns by investing in various types of non risky investment |  |  |  |
| 2.1 |  | e Value of Money, Simple Interest, Periodic Compounding, Streams of ments, Continuous Compounding, Comparing Compounding Methods | [6L] |
| 2.2 |  | ney Market, Zero-Coupon Bonds, Coupon Bonds, Money Market Account | [6L] |
| References: <br> - Mathematics for finance: An Introduction to financial Engineering by Marek Capinski and Tomasz Zastawniak <br> - An Introduction to Mathematical Finance with Applications: Understanding and Building Financial Intuition; Arlie O. Petters, Xiaoying Dong; Springer <br> - Mathematics for Finance: An Introduction to Financial Engineering; by Marek Capiński, Tomasz Zastawniak; Springer |  |  |  |
|  | le 3 | Discrete Time Market Models | [12L] |

## Learning Objectives

The learner should be able to

1. Study various types investment strategies
2. Manage portfolio
3. Study American and European options

## Learning Outcomes

At the end of the module, the learner will be able to

1. Strategize various investment
2. Manage portfolio
3. Solve problems based on American and European options

| 3.1 | Investment Strategies, Fundamental Theorem of Asset Pricing, Portfolio <br> Management, Capital Asset Pricing Model | [6L] |
| :--- | :--- | :--- |
| 3.2 | Put-Call Parity, Bounds on Option Prices, European Options, European and American <br> Calls on Non-Dividend Paying Stock | [6L] |

## References:

- Mathematics for finance: An Introduction to financial Engineering by Marek Capinski and Tomasz Zastawniak
- An Introduction to Mathematical Finance with Applications: Understanding and Building Financial Intuition; Arlie O. Petters, Xiaoying Dong; Springer
- Mathematics for Finance: An Introduction to Financial Engineering;by Marek Capiński, Tomasz Zastawniak; Springer


## T.Y. B. Sc. (Mathematics) SEMESTER V Discipline Specific Practical Course COURSE TITLE: Discipline Specific Elective Practical COURSE CODE: 23US5MTDSP CREDITS - 02]

## Learning Objectives:

The Practical is intended to

1. Solve problems based on the concepts learnt
2. Apply the concepts in various situation

## Learning Outcome:

After the successful completion of the practical, the learner will be able to:

1. Solve problems
2. Apply the results proved
3. Generate examples and counterexamples

## Elective $1 \quad$ Number Theory and its applications I

1.1 Problems based on Twin primes, linear congruence, Complete Residue system modulo m, Reduced residue system modulo m, Euler's Phi Function.
1.2 Problems based on Fermat little Theorem, Euler's generalization of Fermat Little Theorem, Wilson Theorem, Chinese Remainder theorem.
1.3 Problems based on the linear equations $a x+b y=c$. Representation of Prime as sum of two squares, Thue's Lemma, Representation of integers of the type $N^{2} m$ for square free $m$ as sum of two squares.
1.4 Problems based on Pythagorean triples and their solutions.
1.5 Problems based on Quadratic Residues, Euler's Criterion, Legendre symbol and its properties.
1.6 Problems based on Gauss Lemma, Quadratic Reciprocity Law, quadratic congruence equations, the Jacobi Symbol.

## Course Learning Outcome

## After the successful completion of the Course, the learner will be able to:

CLO 1: Write Program to implement Mathematical concepts
CLO 2: Apply Concepts learnt to solve problems

## Learning Objectives:

The learner should be able to:

1. Introduce programming techniques in C
2. Implementation of various mathematical concepts

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Write simple programs
2. Implement mathematical concepts
3. Debug simple programs
4. Predict the output before running the program

| 1.1 | Write a C program that illustrates the concepts of different <br> iterations and conditional requirements. | $[2 \mathrm{~L}]$ |
| :--- | :--- | :--- |
| 1.2 | Write a C program that illustrates the concepts of one- and two- <br> dimensional arrays. | $[4 \mathrm{~L}]$ |
| 1.3 | Write a C program that illustrates the concepts of \#define directives, <br> functions, recursion. | [3L] |

## Reference book

- Let us C by Yashwant Kanetkar, BPB


## Additional References:

- Programming in ANSI C (Third Edition) : E Balagurusamy, TMH
- Programming with C by Byron Gottfried, Schaum's outlines, Mc Graw Hill.
- Mastering Algorithms with C, Kyle Loudon, Shroff Publishers.
- Algorithms in C (Third Edition): Robert Sedgewick, Pearson Education Asia.
- Programming in ANSI C by Ram Kumar, Rakesh Agrawal, TMH.
- Programming with C (Second Edition): Byron S Gottfried (Adapted by Jitender Kumar Chhabra) Schaum's Outlines (TMH)
- Programming with C : K R Venugopal, Sudeep R Prasad TMH Outline Series.


## Elective 3 Application of Graph Theory

[12L]

## Learning Objectives

The learner should be able to:

1. Understand how to apply results
2. Create network
3. Appreciate concepts in planar graphs

## Learning outcomes

The learner will be able to:

1. Construct network
2. Apply results proved
3. Apply Polya's theory to solve some of the problems of Chemistry

| $\mathbf{2 . 1}$ | Construction of network <br> Application of Cayley's formula <br> Application of Menger's theorem | [4L] |
| :--- | :--- | :--- |
| $\mathbf{2 . 2}$ | Problems based on matching, perfect matching and assignment <br> problems | [2L] |
| $\mathbf{2 . 3}$ | Problems based on planarity <br> Chromatic polynomial <br> Timetabling | [4L] |
| 2.4 | Problems based on Polya theory | [2L] |
| Reference books <br> - Graph theory with application by J. A. Bondy and U. S. R. Murty <br> (Freely downloadable) <br> - Graph Theory by Reinhard Diestel Electronic edition Springer Verlag. (Freely downloadable) <br> - Graph theory with application, Narsingh DeoPrentice Hall publication |  |  |

## Elective 4 Financial Mathematics <br> [12L]

## Learning Objectives

The learner should be able to:

1. Appreciate the concepts and how mathematics plays a role in the world of finances
2. Apply principles and process to evaluate the returns of a portfolio

## Learning outcomes

The learner will be able to:

1. Apply Principle of no arbitrage, forward contract and options
2. Evaluate the returns of a portfolio with risky and non risky assets

| 3.1 | Identify problems which are based on no arbitrage principles. <br> Solve problem based on forward contract and options | $[3 L]$ |
| :--- | :--- | :--- |
| 3.2 | Solve problems based on non risky assets | $[3 L]$ |
| 3.3 | Generate a good portfolio of an investor with constraints | $[6 \mathrm{~L}]$ |

## References:

- Mathematics for finance: An Introduction to financial Engineering by Marek Capinski and Tomasz Zastawniak; Springer
- An Introduction to Mathematical Finance with Applications: Understanding and Building Financial Intuition; Arlie O. Petters, Xiaoying Dong; Springer
- Mathematics for Finance: An Introduction to Financial Engineering;by Marek Capiński, Tomasz Zastawniak;


## Students are required to select any one courses

Of the following Generic courses courses, learners can select any one of the courses. The courses having maximum number of votes will be considered for the year and subject to availability of the concerned teacher.

| DSE | Title |
| :--- | :--- |
| GE 1 | Operation Research -I |
| GE 2 | SQL |

## T.Y. B. Sc. (Mathematics) SEMESTER V

## General Elective -I

COURSE TITLE: Operations Research COURSE CODE: 23US5MTGEOR CREDITS - 02]
This is a practical oriented course

After the successful completion of the Course, the learner will be able to:
CLO 1: Prepare mathematical model of real-life problems
CLO 2: Apply Linear Algebra to solve problems of LPP
CLO 3: $\quad$ Generate a method to solve problems of LPP with more than 2 variables

## Module 1 Basics of Operations Research:

## Learning Objectives:

The learner should be able to:

1. Comprehend scope and limitation of Operational research
2. Understand the origin and various development in Operations Research
3. How Linear Algebra is applied to develop Operations Research

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Comprehend scope and limitation of Operational research
2. Apply various stages of development of Operational research
3. Apply Linear Algebra to various problems of Operations Research

| 1.1 | Origin \& Development of Operations Research, Definition and Meaning <br> of Operations Research, Different Phases of an Operational Research <br> Study, Scope and Limitations of Operations Research. | [3L] |
| :--- | :--- | :--- |
| 1.2 | Six Stages of Development of Operations Research <br> I: Observe the problem environment <br> II: Analyze and define the problem | [3L] |


|  | III: Develop a model <br> IV: Select appropriate data input <br> V: Provide a solution and test its reasonableness <br> VI: Implement the solution |  |
| :--- | :--- | :--- |
| 1.3 | Mathematical Modeling of Real-Life Problems. | [4L] |

## Reference books

- G. Hadley: Linear Programming. Narosa, Reprint, 2002.
- Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, 2010.
- A. Ravindran, D. T. Phillips and James J. Solberg: Operations Research- Principles and Practice, John Wiley \& Sons, 2005.
- F.S. Hillier. G.J. Lieberman: Introduction to Operations Research- Concepts and Cases, 9th Edition, Tata Mc-Graw Hill, 2010.


## Module 2 Linear Programming

## Learning Objectives

The learner should be able to:

1. Understand various concepts used in Linear Programming
2. Use Graphical methods to solve Linear Programming problems
3. Use Iso cost line in graphical solutions

## Learning outcomes

The learner will be able to:

1. Formulate the linear programming problems
2. Solve the linear programming problems graphically

| 2.1 | Review of Linear algebra: Solution of a system of Linear Equations, Linear <br> independence and dependence of vectors, Concept of Basis. <br> Basic Feasible solution, Convex sets. Extreme points, Hyperplanes and <br> Halfspaces, Convex cones, Polyhedral sets and cones. | [6L] |
| :--- | :--- | :--- |
| $\mathbf{2 . 2}$ | Linear Programming Problem Formulation, solution by Graphical <br> Method, Graphical Solution of LPP- Bounded region, unbounded region, <br> Use of Iso-Cost/profit lines to find solution | [6L] |

## Reference books

- G. Hadley: Linear Programming. Narosa, Reprint, 2002.
- Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, 2010.
- A. Ravindran, D. T. Phillips and James J. Solberg: Operations Research- Principles and Practice, John Wiley \& Sons, 2005.
- F.S. Hillier. G.J. Lieberman: Introduction to Operations Research- Concepts and Cases, 9th Edition, Tata Mc-Graw Hill, 2010.

The learner should be able to:

1. Understand various concepts applied for solving Linear programming problems by Simplex method and other methods
2. Apply Simplex method to solve Linear Programming problems

## Learning outcomes

The learner will be able to:

1. Understand various concepts applied for solving Linear programming problems by Simplex method and other methods
2. Apply Simplex method to solve Linear Programming problems
3.1 $\quad$ Theory of Simplex Method, slack variables, surplus variables and artificial variables, Simplex Algorithm
Reference books

- G. Hadley: Linear Programming. Narosa, Reprint, 2002.
- Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, 2010.
- A. Ravindran, D. T. Phillips and James J. Solberg: Operations Research- Principles and Practice, John Wiley \& Sons, 2005.
- F.S. Hillier. G.J. Lieberman: Introduction to Operations Research- Concepts and Cases, 9th Edition, Tata Mc-Graw Hill, 2010.


## T.Y. B. Sc. (Mathematics) SEMESTER V

## General Elective -II

COURSE TITLE: SQL
COURSE CODE: 23US5MTGESQL CREDITS - 02]
This is a practical oriented course

After the successful completion of the Course, the learner will be able to:
CLO 1: Apply commands to fire a query

## Module 1 SQL <br> [36L]

## Learning Objectives:

The learner should be able to:

1. Understand the commands of SQL

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Fire queries to obtain the desired output

| 1.1 | SQL Overview, SQL commands, Concepts of RDBMS, field, record, <br> column, SQL constraints, NULL, NOT NULL constraints, default <br> constraints, DROP, Primary key, creating and deleting primary key, <br> Foreign key, drop a foreign key, CHECK constraints, DROP a CHECK <br> constraint, INDEX, DROP INDEX constraint | [10L] |
| :--- | :--- | :--- |
| 1.2 | SELECT statement, arithmetic operators, logical operators, comparison <br> operators, USE statement, CREATE and DROP a DATABASE, creating, <br> listing and dropping a table, Truncate table, ALTER table, INSERT INTO, <br> UPDATE, DELETE | [13L] |
| 1.3 | DISTINCT clause, WHERE clause, AND/OR IN clause, BETWEEN clause, <br> LIKE clause, ORDERBY clause, GROUPBY clause, COUNT, HAVING clause | [13L] |
| Reference books <br> - SQL Tutorial |  |  |

## T.Y. B. Sc. (Mathematics) SEMESTER V <br> Skill Enhancement Course <br> COURSE TITLE: LaTeX <br> COURSE CODE: 23US6MTSELTX CREDITS - 02]

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Use LaTeX for writing mathematical articles and books

CLO 2: Make presentation using LaTeX

| Module 1 | LaTeX | [18L] |
| :--- | :--- | :--- |

## Learning Objectives

The learner should be able to:

1. Use LaTeX for documentation of Mathematical work

## Learning Outcomes

After the successful completion of the module, the learner will be able to:

1. Write mathematical expressions using LaTeX
2. Draw figures using LaTeX

| $\mathbf{1 . 1}$ | Documentation, letter writing | [6L] |
| :--- | :--- | :--- |
| $\mathbf{1 . 2}$ | Tables, Matrices, Mathematical functions | [6L] |
| 1.3 | Drawing a diagram, Labelling | [6L] |

## Reference books

- LaTeX Manual


## Module 2 Beamer

## Learning Objectives

The learner should be able to:

1. Use LaTeX to make presentation

## Learning outcomes

The learner will be able to:

1. Make presentation using LaTeX

| 2.1 | Documentation and compiling beamer presentation | [9L] |
| :--- | :--- | :--- |
| 2.2 | Preparing and formatting slides | [9L] |
| Reference books  <br> $\bullet$ LaTeX manual |  |  |

# T.Y. B. Sc. (Mathematics) SEMESTER VI <br> Core Course-I 

## COURSE TITLE: Multivariate Integral Calculus COURSE CODE: 23US6MTCC1MIC CREDITS - 02

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Evaluation of double and triple integrals
CLO 2: Apply Green's theorem
CLO 3: Apply Stokes and Divergence theorem

## Module 1 Double and Triple integrals

## Learning Objectives:

The learner should be able to:

1. Solve problems on double and triple integrals
2. Apply the concept to real life situations

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Appreciate the concepts of double and triple integrals
2. Prove Fubini's Theorem and other results
3. Solve problems based on Fubini's Theorem and change of variables formula
4. Compute area, volume, the center of gravity and moments of inertia

| 1.1 | Review of Riemann integration. <br> Definition of double (resp: triple) integral of a function and bounded on a <br> rectangle (resp:box). | [2L] |
| :--- | :--- | :--- |
| 1.2 | Fubini's Theorem over rectangles and on any closed bounded region, <br> Iterated Integrals. <br> Basic properties of double and triple integrals proved using the Fubini's <br> theorem such as <br> (i) Integrability of the sums, scalar multiples, products, and (under suitable <br> conditions) quotients of integrable functions. <br> (ii) Integrability of continuous functions. <br> (iii) Domain additivity of the integral. Integrability and the integral over <br> arbitrary bounded domains. | [4L] |
| 1.3 | Change of variables formula. Use of Polar coordinates in evaluation of <br> double integrals. <br> Cylindrical and spherical coordinates, and triple integration using these <br> coordinates. | [3L] |


|  | Simple examples using other changes of variables in double integration. |  |
| :--- | :--- | :--- |
| 1.4 | Applications to finding area and volume, the center of gravity and moments <br> of inertia. | [3L] |
| Reference books |  |  |

- T. M. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- James Stewart; Multivariable Calculus


## Additional Reference books:

- D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989
- Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis (12th Edition, 1979), S. Chand and Co
- M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., SpringerVerlag, New York, 1995


## Module 2 Line Integrals

[12L]

## Learning Objectives

The learner should be able to:

1. Parametrize curves
2. Apply tools developed to solve problems
3. Appreciate the independence of path in evaluating line integral under a conservative force field

## Learning outcomes

The learner will be able to:

1. Find parametrization of a given curve
2. Understand the concept of line integrals for scalar and vector functions
3. Solve problems using the above
4. Apply the Green's Theorem
5. Solve problems on conservative force fields

| 2.1 | Parametrization of a curve in the plane or in space. Line integral of a scalar <br> valued function. Finding length of a curve | [3L] |
| :--- | :--- | :--- |
| 2.2 | Line integral of a vector valued function. Application in computation of <br> work done by a force and flow in a medium. | [3L] |
| 2.3 | Green's theorem and its use in evaluation of line integrals, the area of <br> regions enclosed by a simple closed curve | [3L] |
| 2.4 | Conservative fields and path independence of the line integral, its <br> equivalence with the existence of potential function. Applications | [3L] |

## Reference books

- T. M. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- James Stewart; Multivariable Calculus
- Jerrold E. Marsden, Anthony Tromba Vector Calculus; Basic multivariable calculus


## Additional Reference books:

- G.B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1992
- D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989
- Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis (12th Edition, 1979), S. Chand and Co
- M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., SpringerVerlag, New York, 1995


## Module 3

Surface Integrals and Volume Integrals

## [12L]

## Learning Objectives

The learner should be able to:

1. Parametrize surfaces
2. Find a normal vector to the surface
3. Understand the concept of surface integrals
4. Prove Stoke's Theorem and Divergence Theorem
5. Study other applications of surface integrals

## Learning outcomes

The learner will be able to:

1. Find parametrization for a given surface
2. Find a normal vector to the surface using parametrization
3. Apply the idea of surface integrals and solve problems
4. Prove Stoke's Theorem and Divergence Theorem
5. Apply Stoke's Theorem and Divergence Theorem

| 3.1 | Parametrization of surfaces. Finding area of a surface using <br> fundamental vector product | [3L] |
| :--- | :--- | :--- |
| 3.2 | Definition of surface integrals of scalar valued as well as of vector <br> valued functions defined on a surface | [3L] |
| 3.3 | Curl and divergence of a vector field. Elementary identities involving <br> gradient, curl and divergence. <br> Stokes Theorem (proof assuming the general form of Green's | [6L] |


|  | Theorem). Examples. Gauss Divergence Theorem (proof only in the case <br> of cubical domains). Examples |  |
| :--- | :--- | :--- |
| Reference books |  |  |

- T. M. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- James Stewart; Multivariable Calculus
- Jerrold E. Marsden, Anthony Tromba Vector Calculus; Basic multivariable calculus


## Additional Reference books:

- G.B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1992
- D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989
- Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis (12th Edition, 1979), S. Chand and Co
- M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., SpringerVerlag, New York, 1995


## T.Y. B. Sc. (Mathematics) SEMESTER VI <br> Core Course- II <br> COURSE TITLE: Algebra-III COURSE CODE: 23US6MTCC2ALG [CREDITS - 02]



| 1.4 | External direct product and its properties, order of an element in external <br> direct product, criteria for external direct product to be cyclic. | [3L] |
| :--- | :--- | :--- |
| 1.5 | Classification of groups of order up to 7 | [2L] |

## Reference books

- I.N.Herstein - Topics in Algebra
- N.S. Gopalkrishnan - University Algebra
- J.B.Fraleigh- A first course in abstract algebra
- M. Artin - Algebra
- J. Gallion - contemporary abstract algebra


## Additional Reference books:

- D. Dummit and R. Foote- Abstract algebra
- Bhattacharya Jain Nagpal - Abstract Algebra


## Module 2 Rings and Fields

## Learning Objectives

The learner should be able to:

1. Prove all results related to rings, subrings, ideals, and ring homomorphism
2. Identify special elements in a ring
3. Study prime and maximal ideals
4. Appreciate ring homorphism

## Learning outcomes

The learner will be able to:

1. Prove all results related to rings, subrings, ideals, and ring homomorphism
2. Find zero divisors, units, idempotent elements, nilpotent elements in a ring
3. Classify prime and maximal ideals
4. Construct quotient field
5. Solve problems based on rings and ring homomorphism

| $\mathbf{2 . 1}$ | Definition of Rings (with unity), zero divisors, Integral domain, Division rings <br> and fields, various examples including polynomial rings | [3L] |
| :--- | :--- | :--- |
| $\mathbf{2 . 2}$ | Units in a ring, idempotent and nilpotent elements, the multiplicative group <br> of units in a ring, characteristic of a ring | [2L] |


| $\mathbf{2 . 3}$ | Subrings and ideals, prime and maximal ideals | [2L] |
| :--- | :--- | :--- |
| 2.4 | Quotient rings, characterization in terms of prime and maximal ideals, <br> construction of quotient field, correspondence theorem | [3L] |
| 2.5 | Ring homomorphism and isomorphism, kernel, and properties | [2L] |

## Reference books

- I.N.Herstein - Topics in Algebra
- N.S. Gopalkrishnan - University Algebra
- J.B.Fraleigh- A first course in abstract algebra
- M. Artin - Algebra
- J. Gallion - contemporary abstract algebra


## Additional Reference books:

- D. Dummit and R. Foote- Abstract algebra
- Bhattacharya Jain Nagpal - Abstract Algebra

\section*{| Module 3 | Factoriz |
| :--- | :--- |
| Learning Objectives |  |}

The learner should be able to:

1. Prove results related to various concepts of factorization in rings
2. Verify irreducibility and prime elements in a ring
3. Appreciate prime and maximal ideals
4. Establish the properties of Euclidean domain, Principal ideal domain and

Unique factorization domain

## Learning outcomes

The learner will be able to:

1. Prove results related to various concepts of factorization in rings
2. Classify irreducibility and prime elements in a ring
3. Find prime and maximal ideals in a polynomial ring
4. Classify the rings as Euclidean domain, Principal ideal domain and Unique factorization domain
5. Solve problems associated with factorization of rings
3.1

Polynomial rings, properties

| $\mathbf{3 . 2}$ | Irreducible and prime elements, ideal generated by irreducible and prime <br> elements, prime and maximal ideals in a polynomial ring | [4L] |
| :--- | :--- | :--- |
| 3.3 | Euclidean domain (ED), principal ideal domain (PID) and unique factorization <br> domain (UFD) | [3L] |
| 3.4 | Every ED is PID, every PID is UFD | [3L] |

## Reference books

- I.N.Herstein - Topics in Algebra
- N.S. Gopalkrishnan - University Algebra
- J.B.Fraleigh- A first course in abstract algebra
- M. Artin - Algebra
- J. Gallion - contemporary abstract algebra


## Additional Reference books:

- D. Dummit and R. Foote- Abstract algebra
- Bhattacharya Jain Nagpal - Abstract Algebra


# T.Y. B. Sc. (Mathematics) SEMESTER VI <br> Core Course- III 

## COURSE TITLE: Topology of Metric Spaces II COURSE CODE: 23US6MTCC4TMS2 CREDITS - 02

| Cours | arning Outcome |  |
| :---: | :---: | :---: |
| After th CLO 1 <br> CLO 2 <br> CLO 3 | e successful completion of the course, the learner will be able to: <br> Appreciate various characterizations of continuous functions and uniformly continuous functions. <br> Understand Compact space <br> Understand connectedness. |  |
| Modu le 1 | Continuous functions defined on metric spaces | [12L] |
| Learning Objectives: <br> The aim of the module is to: <br> 1. Study various characterizations of continuous functions over a metric space <br> 2. Establish unique extension of a uniformly continuous function over a dense subset of a metric with complete codomain |  |  |
| Learning Outcome: <br> After the successful completion of the module, the learner will be able to: <br> 1. Prove results based on continuous functions on a metric space <br> 2. Solve problems using $\varepsilon-\delta$ definition of continuity and sequential criterion <br> 3. Establish unique extension of a uniformly continuous function over a dense subset of a metric with complete codomain |  |  |
| 1.1 | $\varepsilon-\delta$ definition of continuity at a point of a function from one metric space to another. Examples. <br> Algebra of continuous real valued functions. | [3L] |
| 1.2 | Characterization of continuity at a point in terms of sequences. Characterization in terms of inverse image of open sets and closed sets. | [3L] |


| 1.3 | Uniform continuity in a metric space, definition and examples (emphasis on $\mathbb{R}$ ). Contraction mapping is uniformly continuous. Image of a Cauchy sequence under a uniformly continuous function forms a Cauchy sequence. <br> Set of real or complex valued uniformly continuous functions defined on a metric space forms a vector space. Product of two uniformly continuous functions need not be uniformly continuous. | [3L] |
| :---: | :---: | :---: |
| 1.4 | Continuous image of a complete metric space need not be a complete metric space. <br> If $A$ is a dense subset of a metric space $X$ and $f$ is a uniformly continuous function from $A$ to $Y$, where $Y$ is a complete metric space, then $f$ can be uniquely extended to a continuous function on taking values in. As a consequence of this, we can define irrational powers of positive real numbers. | [4L] |
| Reference books <br> - Metric Spaces P. K. Jain and Khalid Ahmad, Narosa Publications <br> - Copson, E. T. (1996) Metric Spaces. Universal Book Stall, New Delhi. Additional Reference books: <br> - Topology of Metric spaces; Kumaresan, S. (2005) |  |  |
| Mod | 2 Compactness | [12L] |
| Learning Objectives <br> The aim of the module is to: <br> 1. Study compact metric space <br> 2. Appreciate characterisations of compactness |  |  |
| Learning Outcomes <br> The learner will be able to: <br> 1. Prove results associated with compact metric space <br> 2. Solve problems based on results proved |  |  |
| 2.1 | Definition of a compact set in a metric space using an open cover Examples of compact sets in different metric spaces with emphasis on $\mathbb{R}, \mathbb{R}^{2}, \mathbb{R}^{3}$ | [3L] |


| 2.2 | Properties of compact sets such as a compact set is closed and bounded. Converse is false. <br> Every infinite bounded subset of a compact metric space has a limit point. As a consequence, every compact Metric space is complete. Closed subset of a compact space is compact as well as complete. Heine Borel theorem: Every subset of Euclidean metric space $\mathbb{R}^{n}$ is compact if and only if it is closed and bounded. | [5L] |
| :---: | :---: | :---: |
| 2.3 | A continuous function on a compact set is uniformly continuous. Continuous image of a compact set is compact. <br> Fixed point theorem for a contractive map defined on a compact space. | [2L] |
| 2.4 | Totally bounded sets. Bolzano-Weierstrass property, sequentially compactness property. <br> Characterization of compact sets in: The equivalent statements for a subset of metric space to be compact. | [2L] |
| Reference books <br> - Metric Spaces P. K. Jain and Khalid Ahmad, Narosa Publications <br> - Copson, E. T. (1996) Metric Spaces. Universal Book Stall, New Delhi. Additional Reference books: <br> - Kumaresan, S. (2005) Topology of Metric spaces |  |  |
| Mod | e 3 Connected metric spaces | [9L] |
| Learning Objectives <br> The aim of the module is to: <br> 1. Study Connected subsets of a metric space <br> 2. Differentiate between connected sets and path connected sets |  |  |
| Learning Outcomes <br> The learner will be able to: <br> 1. Prove results associated with connected sets and path connected sets <br> 2. Apply results proved |  |  |
| 3.1 | Definition and examples of Separated sets in a metric space, Disconnected sets, Connected sets. | [2L] |
| 3.2 | Connected subset of a metric space is open as well as closed. Definition of component of a connected space. | [2L] |


| $\mathbf{3 . 3}$ | A continuous image of a connected set is connected. <br> A subset of $\mathbb{R}$ is connected if and only if it is either an interval or a <br> singleton. Intermediate value property of real valued functions defined <br> on connected metric spaces. <br> Characterization of a connected space, namely a metric space $X$ is <br> connected if and only if every continuous function from $X$ to $\{-1,1\}$ is a <br> constant function. | [5L] |
| :--- | :--- | :--- |
| 3.4 | Path connectedness in $\mathbb{R}^{n}$, definitions and examples. <br> A path connected subset of $\mathbb{R}^{n}$ is connected. <br> An example of a connected subset of $\mathbb{R}^{2}$ which is not path connected. |  |

## Reference books

- Metric Spaces P. K. Jain and Khalid Ahmad, Narosa Publications
- Copson, E. T. (1996) Metric Spaces. Universal Book Stall, New Delhi. Additional Reference books:
- Kumaresan, S. (2005) Topology of Metric spaces


# T.Y. B. Sc. (Mathematics) SEMESTER VI 

## Core Course- IV

## COURSE TITLE: Complex Analysis

## COURSE CODE: 23US6MTCC4CA CREDITS-02

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Compute derivatives of complex valued functions
CLO 2: Find integration of complex valued of functions
CLO 3: Identify the singularities

## Learning Objectives

The learner should be able to:

1. Understand the concept of differentiability of a complex function using CauchyRiemann equations
2. Find the derivative of a complex function if it exists

## Learning Outcomes

After the successful completion of the module, the learner will be able to:

1. Decide about differentiability of a complex function using Cauchy-Riemann equations
2. Solve problems to find the derivative of a complex function if it exists
3. Apply Cauchy-Riemann equation for finding harmonic Conjugates
4. Prove results such as Cauchy Riemann Equations and related theory

| 1.1 | Review of complex number, Algebraic properties of complex numbers, <br> Complex Conjugate, Exponential form, Roots of complex numbers. <br> (No questions to be asked in semester end examination) <br> Metric topology of Complex Numbers: Neighbourhood of a point. Open <br> sets and closed sets, limit point, Boundary of a set, <br> Sequences and series of complex numbers, converges in C if and only if <br> both real and imaginary part converges in R. | [3L] |
| :--- | :--- | :--- |
| 1.2 | Function of complex Variable, Limits and Theorem on Limits (connection <br> between limits of a function of complex variable and limits of real valued <br> functions of two real variables) <br> Continuity of a function, Derivatives, Differentiation Rules and Examples | [3L] |
| 1.3 | Cauchy-Riemann Equations, Sufficient Conditions for differentiability, Polar <br> Coordinates, Analytic functions (if it is differentiable in neighbourhood of <br> point) f,g analytic then $f+g, f-g, f . g$ and $f / g$ are analytic, chain rule of | [3L] |


|  | differentiation. |  |
| :---: | :---: | :---: |
| 1.4 | Theorem: If $f(z)=0$ everywhere in a domain D , then $\mathrm{f}(\mathrm{z})$ must be constant throughout D. <br> Harmonic functions, Harmonic conjugate and examples | [3L] |
| References: <br> - J.W. Brown and R.V. Churchill, Complex Variables and Applications, International Student Edition, 2009. (Eighth Edition). <br> - S. Ponnusamy, Complex Analysis, Second Edition (Narosa) <br> Additional Reference books: <br> - A.R. Shastri, An Introduction to Complex Analysis, (MacMillan). <br> - John B. Conway Functions of one Complex variable, Second Edition, Springer-Verlag |  |  |
| Mod | e 2 Elementary Functions \& Integral along a curve | 12L |
| Learning Objectives <br> The learner should be able to: <br> 1. Study Exponential function, trigonometric and hyperbolic functions <br> 2. Understand logarithm of complex number and observe how it is different from logarithm of real numbers <br> 3. Find the contour integral of a complex function |  |  |
| Learning Outcomes <br> After the successful completion of the module, the learner will be able to: <br> 1. Establish the periodicity of Exponential function <br> 2. Find the principal and general branch of logarithm of a complex number <br> 3. Evaluate the contour integral of a function <br> 4. Apply Cauchy Integral formula to find Contour integral |  |  |
| 2.1 | The Exponential function, Periodicity of the Exponential function. The Logarithmic function, Branch of logarithms, Some identities involving logarithms, Trigonometric functions, Hyperbolic functions, | [4L] |
| 2.2 | Definition of a Line integral of a function along a curve. Contours, Contour integral, Examples. <br> Upper bounds for Moduli of contour integrals | [2L] |
| 2.3 | Anti-derivatives, Examples <br> Cauchy Groursat's Theorem (without proof), Simply and multiply Connected domains | [3L] |
| 2.4 | Evaluation of line integral $\int f(z) d z$ over $\left\|z-z_{0}\right\|=r$, Cauchy integral formula (with proof), An Extension of the Cauchy Integral formula. <br> Liouville's Theorem and Fundamental Theorem of Algebra. | [3L] |

References:

- J.W. Brown and R.V. Churchill, Complex Variables and Applications, International Student Edition, 2009. (Eighth Edition).
- S. Ponnusamy, Complex Analysis, Second Edition (Narosa)


## Additional Reference books:

- A.R. Shastri, An Introduction to Complex Analysis, (MacMillan).

John B. Conway Functions of one Complex variable, Second Edition, Springer-Verlag Module 3 Power Series and Singularities

## Learning Objectives

The learner should be able to:

1. Introduce power series of complex numbers
2. Find Taylor's series, Laurent series of a complex function
3. Classify singularities
4. Obtain residues

## Learning Outcomes

After the successful completion of the module, the learner will be able to:

1. Find radius of convergence of a power series
2. Find Taylor's series and Laurent series of a complex function
3. Categorize singularities as removable or isolated singularity
4. Obtain residues and poles of Complex function

| 3.1 | Power series of complex numbers definition and examples <br> Radius of convergences, disc of convergence, uniqueness of series <br> representation, examples | [2L] |
| :--- | :--- | :--- |
| 3.2 | Taylor's series, Taylor's theorem for analytic function, Laurent series <br> (without proof), example | [3L] |
| 3.3 | Isolated singular points, Types of isolated singular points, Examples | [3L] |
| 3.4 | Residues, Cauchy residue theorem, Residue at Infinity, residues at poles, <br> zeros of analytic functions | [4L] |

References:

- J.W. Brown and R.V. Churchill, Complex Variables and Applications, International Student Edition, 2009. (Eighth Edition).
- S. Ponnusamy, Complex Analysis, Second Edition (Narosa)

Additional Reference books:

- A.R. Shastri, An Introduction to Complex Analysis, (MacMillan).
- John B. Conway Functions of one Complex variable, Second Edition, Springer-Verlag


## T. Y. B. Sc. (Mathematics)

SEMESTER VI - Practical - I

## Learning Objectives:

The Practical is intended to

1. Solve problems based on the concepts learnt
2. Apply the concepts in various situation

## Learning Outcome:

After the successful completion of the practical, the learner will be able to:

1. Solve problems
2. Apply the results proved
3. Generate examples and counterexamples

| Module I | Multivariate Integral Calculus |
| :--- | :--- |

1.1 Sketching region of integration for double and triple integral. Computation using iterated integrals. Change of order of integration.
1.2 Change of variables. Use of polar, cylindrical and spherical coordinates. Finding center of mass, moment of inertia.
1.3 Parametrization of a curve in two or three dimensions. Computation of line integral for scalar valued as well as vector valued functions. Finding length of a curve. Finding work done by a force, flow and flux.
1.4 Use of Green's theorem. Integration by change of parametrization. Computation of area of a region enclosed by a simple closed curve Conservative functions, potential function for a conservative function. Verification of path independence in case of a conservative functions
1.5 Parametrization of surfaces. Computation of vectors normal to the given surface. Simple problems on computation of surface integrals. Using change of variables
1.6 Divergence and curl. Problems on use of Stoke's theorem and Divergence theorem. Verification of Stoke's theorem and Divergence theorem

| Module 2 | Group theory and Ring theory |
| :--- | :--- |

### 2.1.Normal subgroups, quotient groups

2.2.Cayley's thm and external direct product
2.3.Ring, subring Integral domain
2.4. Homomorphism and isomorphism of rings
2.5. Polynomial rings
2.6.Divisibility, prime and maximal ideals

T. Y. B. Sc. (Mathematics)<br>SEMESTER VI - Practical - II<br>COURSE CODE: 23US6MTCCP2 Credit- 02

## Learning Objectives:

The Practical is intended to

1. Solve problems based on the concepts learnt
2. Apply the concepts in various situation

## Learning Outcome:

After the successful completion of the practical, the learner will be able to:

1. Solve problems
2. Apply the results proved
3. Generate examples and counterexamples

| Module I | Topology of Metric Spaces II |
| :--- | :--- |

1.1 Problems on Continuous functions and sequential continuity.
1.2 Problems on Uniform continuity.
1.3 Problems based on the definition of compact sets. Heine Borel theorem.
1.4 Problems based on sequential compactness, fixed point theorem. Image of compact sets under continuous map.
1.5 Problems based on Separated sets and connected spaces. Components
1.6 Problems on path connected sets. Intermediate value property.

| Module II | Complex Analysis |
| :--- | :--- |

2.1 Problems on Function of complex Variable, Limits and Theorem on Limits (connection between limits of a function of complex variable and limits of real valued functions of two real variables), Continuity of a function, Derivatives, Differentiation Rules .
2.2 Problems on Cauchy-Riemann Equations, Sufficient Conditions for Differentiability, Polar Coordinates, Analytic functions, Harmonic functions, Harmonic conjugate.
2.3 Problems on The Exponential function, Periodicity of the Exponential function, the Logarithmic function, Branch of logarithms, Trigonometric functions, Hyperbolic functions, Line integral of a function along a curve, Contours, Contour integral.
2.4 Problems on Anti-derivatives, Cauchy Groursat's Theorem, Simply and multiply Connected domains. Evaluation of line integral $\int f(z)$ dz over $|z-z 0|=r$. Problems on Cauchy integral formula, An Extension of the Cauchy Integral formula, Liouville's Theorem.
2.5 Problems on Radius of convergences, disc of convergence, uniqueness of series representation, Taylor's series, Taylor's theorem for analytic function, Laurent series
2.6 Problems on Isolated singular points, Types of isolated singular points, Residues, Cauchy residue theorem, Residue at Infinity, residues at poles, zeros of analytic functions

## References:

1. S. Ponnusamy, Complex Analysis, Second Edition (Narosa)
2. J.W. Brown and R.V. Churchill, Complex Variables and Applications, International Student Edition, 2009. (Eighth Edition).
3. Complex Function theory; Donald Sarason; Hindustan book agency

TYBSC ( MATHEMATICS) SEMESTER VI
Discipline Specific Elective Courses
Students are required to select any two courses

| DSE | Title |
| :--- | :--- |
| DSE 1 | Number Theory and its applications II |
| DSE 2 | JAVA Programming |
| DSE 3 | Combinatorics |
| DSE 4 | Game Theory |
| DSE 5 | Mathematical elements in computer graphics |

Discipline Specific Course-I
COURSE TITLE: Number Theory and its applications II
COURSE CODE: 23US6MTDS1NTA CREDITS - 02]

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Represent a real number with simple continued fraction
CLO 2: Find solutions of Pell's equations
CLO 3: Use classical cryptosystem to encrypt and decrypt a message

| Module 1 | Continued Fractions | [12L] |
| :--- | :--- | :--- |

## Learning Objectives

The learner should be able to:

1. Understand the concept of continued fraction
2. Apply theory of continued fraction to solve problems

## Learning Outcomes

The learner will be able to:

1. Prove results related to finite and infinite continued fractions
2. Approximate an irrational with a rational with desired accuracy.
3. Solve Diophantine a equation using continued fraction

| 1.1 | Definition of finite continued fraction. Two different representations of a rational <br> number as a finite simple continued fraction. Value of a finite continued fraction is <br> always rational. Solving linear Diophantine equation using Continued Fraction. | [4] |
| :--- | :--- | :--- |
| 1.2 | $K^{\text {th }}$ convergent $\left(C_{k}\right)$ of a continued fraction. Representation of $C_{k}$ as $p_{k} / q_{k}$ The <br> convergents with even subscript forms a strictly increasing sequence and | [4] |


|  | convergents with odd subscript forms a strictly decreasing sequence. |  |  |
| :---: | :---: | :---: | :---: |
| 1.3 | Definition of infinite continued fraction. Representation of an irrational number as an infinite simple continued fraction. Value of an infinite continued fraction is always irrational. Every irrational number has a unique representation as an infinite continued fraction. Rational approximation of an irrational number. If $1 \leq b \leq q_{n}$ then $p_{n} / q_{n}$ is better rational approximation for irrational number $x$ than any rational number $a / b$. |  | [4] |
| Reference books <br> - David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw Hill <br> - H. Zuckerman and H. Montgomery. Elementary number theory. John Wiley \& Sons. Inc. <br> Additional Reference books <br> - G. H. Hardy, and E.M. Wright, An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press |  |  |  |
|  |  | Pell's Equations, Arithmetic Functions and Special Numbers |  |
| Learning Objectives <br> The learner should be able to: <br> 1. Solve Pell's equations <br> 2. Prove certain properties of some Special Numbers |  |  |  |

## Learning outcomes

The learner will be able to:

1. Find solutions of Pell's equations
2. Compute arithmetic functions $d(n)$ (or $\tau(n)), \sigma(n)$
3. Prove results related to special numbers mentioned.

| 2.1 | Pell's equation $x^{2}-d y^{2}=n$, where $d$ is not a square of an integer. Solutions of <br> Pell's equation. (The proofs of convergence theorems to be omitted) | [4L] |
| :--- | :--- | :--- |
| 2.2 | Arithmetic functions of number theory: $d(n)$ (or $\tau(n)), \sigma(n)$ and their properties. <br> $\mu(n)$ and the Mobius inversion formula | [3L] |
| 2.3 | Special numbers: Fermats numbers, Perfect numbers, Amicable numbers, Pseudo <br> primes, Carmichael numbers. Algebraic and transcendental numbers. Algebraic <br> integers, minimal polynomial of algebraic numbers. | [5L] |

## Reference books

- David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw Hill
- H. Zuckerman and H. Montgomery. Elementary number theory. John Wiley \& Sons. Inc.


## Additional Reference books

G. H. Hardy, and E.M. Wright, An Introduction to the Theory of Numbers. Low priced edition.

The English Language Book Society and Oxford University Press

## Learning Objectives

The learner should be able to:

1. Use some classical methods to encrypt and decrypt a message

## Learning Outcomes

The learner will be able to:

1. Encrypt and decrypt a message using classical cryptosystem.

| 3.1 | Classical Cryptosystems, Shift cipher, affine cipher, Vigenere cipher. | [4] |
| :--- | :--- | :--- |
| 3.2 | Play Fair cipher, ADFGX cipher, Block Cipher. | [4] |
| 3.3 | Public Key cryptosystem, RSA algorithm | [4] |

Reference books

- David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw Hill
- Wade Trappe, Lawrence C Washington Introduction to Cryptography with Coding Theory.


## Additional Reference books

- H. Zuckerman and H. Montgomery. Elementary number theory. John Wiley \& Sons. Inc.
- G. H. Hardy, and E.M. Wright, An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981


## T.Y. B. Sc. (Mathematics) SEMESTER VI <br> Discipline Specific Course- II COURSE TITLE: <br> COURSE CODE: 23US6MTDSE2JP CREDITS - 02]

## Course Learning Outcome

CLO 1: Write programs using basic tools available in Java including built in functions
CLO 2: Apply principles of inheritance and write programs
CLO 3: Create various geometrical figures using Applet

## Module 1 Java application program, an introduction

## Learning Objectives:

The learner should be able to:

1. Understand the difference in structured programming and object-oriented programming, Features of OOPs.
2. Accept data from the command prompt.
3. Create classes and objects.
4. Understand various data types and their conversion to another data type.
5. Understand Some special methods its uses
6. Understand difference in Java's approach to arrays as compared to C.

## Learning Outcome:

After the successful completion of the module, the learner will be able to:

1. Create classes with or without instance variables/ methods
2. Typecast different types of data as per requirement
3. Create arrays and access them

| 1.1 | Object-Oriented approach: Comparison between structured and object-oriented <br> approach. Features of object-orientations: Abstraction, Inheritance, <br> Encapsulation and Polymorphism. Concept of package. Integer class method: <br> parseInt(). | [2L] |
| :---: | :--- | :--- |
| 1.2 | Introduction: History of Java, Java features, different types of Java programs, <br> Differentiate Java with C. Java Virtual Machine. | [4L] |
| 1.3 | Java Basics: Variables and data types, declaring variables, literals: numeric, <br> Boolean, character and String literals, keywords, type conversion and casting. <br> Standard default values. Java Operators, Loops and Controls. <br> Classes: Defining a class, creating instance and class members: creating object of <br> a class; accessing instance variables of a class; creating method; naming method <br> of a class; accessing method of a class; 'this' keyword, constructor Basic | [3L] |


|  | Constructor; parameterized constructor; calling another constructor. Finalizer <br> method (only concepts) |  |
| :--- | :--- | :--- |
| 1.4 | Arrays: one and two-dimensional array, declaring array variables, creating array <br> objects, accessing array elements. | [3L] |

## Main Reference:

- Java The Complete Reference, 8th Edition, Herbert Schildt, Tata McGraw Hill


## Additional References:

- Programming with Java: A Primer 4th Edition by E. Balagurusamy, Tata McGraw Hill.
- Eric Jendrock, Jennifer Ball, D Carson and others, The Java EE 5 Tutorial, Pearson Education, Third Edition, 2003.
- Ivan Bayross, Web Enabled Commercial Applications Development Using Java 2, BPB Publications, Revised Edition, 2006
- Joe Wigglesworth and Paula McMillan, Java Programming: Advanced Topics, Thomson Course Technology (SPD), Third Edition, 2004.
- The Java Tutorials of Sun Microsystems Inc. http://docs.oracle.com/javase/tutorial


## Module 2 <br> JAVA: INHERITANCE <br> [12L]

## Learning Objectives

The learner should be able to:

1. Introduce Inheritance, its scope and limitations.
2. Implement inheritance for various mathematical concepts.

## Learning outcomes

The learner will be able to:

1. Write programs involving inheritance
2. Overload methods for various instances
3. Override methods for various instances

| 2.1 | Inheritance: Various types of inheritance, super and subclasses, keywords- <br> 'extends'; 'super', final. | [4L] |
| :--- | :--- | :--- |
| 2.2 | overloading methods | [2L] |
| $\mathbf{2 . 3}$ | overriding methods | [4L] |

## Main Reference:

- Java The Complete Reference, 8th Edition, Herbert Schildt, Tata McGraw Hill


## Additional References:

- Programming with Java: A Primer 4th Edition by E. Balagurusamy, Tata McGraw Hill.
- Eric Jendrock, Jennifer Ball, D Carson and others, The Java EE 5 Tutorial, Pearson Education, Third Edition, 2003.
- Ivan Bayross, Web Enabled Commercial Applications Development Using Java 2, BPB Publications, Revised Edition, 2006
- Joe Wigglesworth and Paula McMillan, Java Programming: Advanced Topics, Thomson Course Technology (SPD), Third Edition, 2004.
- The Java Tutorials of Sun Microsystems Inc. http://docs.oracle.com/javase/tutorial


## Module 3 JAVA: APPLETS <br> [12L]

## Learning Objectives

The learner should be able to:

1. The aim of this module is to introduce concepts of Applet
2. Create animation for various effects.

## Learning outcomes

The learner will be able to:

1. Draw simple and complex geometric objects
2. Create animation using simple geometric structures

| 3.1 | JAVA Applets: Difference of applet and application, creating applets, applet life <br> cycle. | [3L] |
| :--- | :--- | :--- |
| 3.2 | Graphics, Fonts and Color: The graphics class, painting, repainting and updating <br> an applet, sizing graphics. Font class, draw graphical figures - lines and <br> rectangle, circle and ellipse, drawing arcs, drawing polygons. Working with <br> Colors: Color methods, setting the paint mode. | [3L] |

## Main Reference:

- Java The Complete Reference, 8th Edition, Herbert Schildt, Tata McGraw Hill


## Additional References:

- Programming with Java: A Primer 4th Edition by E. Balagurusamy, Tata McGraw Hill.
- Eric Jendrock, Jennifer Ball, D Carson and others, The Java EE 5 Tutorial, Pearson Education, Third Edition, 2003.
- Ivan Bayross, Web Enabled Commercial Applications Development Using Java 2, BPB Publications, Revised Edition, 2006
- Joe Wigglesworth and Paula McMillan, Java Programming: Advanced Topics, Thomson Course Technology (SPD), Third Edition, 2004.
- The Java Tutorials of Sun Microsystems Inc. http://docs.oracle.com/javase/tutorial


## TYBSC ( MATHEMATICS) SEMESTER VI <br> DSE Course- III <br> COURSE TITLE: COMBINATORICS <br> COURSE CODE: 23US6MTDS3CMB

| Course Learning Outcomes |  |
| :---: | :---: |
| After the successful completion of the Course, the learner will be able to: <br> CLO 1: Count all possible distribution of objects in boxes either identical or no identical <br> CLO 2: Apply principle of inclusion and exclusion, pigeon hole to solve problem <br> CLO 3: Solve recurrence relations |  |
| Module 1 Basic Counting | [12L] |
| Learning Objectives <br> The learner should be able to: <br> 1. Study basic counting techniques, <br> 2. Appreciate distribution with different criterion <br> 3. Prove results related to the Stirling number and other theories |  |
| Learning Outcomes <br> After successful completion of the course the learner will be able to: <br> 1. Count all possible distribution when the objects and boxes are identical and non-identical <br> 2. Prove results related to Stirling number and other theories using combinatorial arguments <br> 3. Solve problems based on distribution of objects, Stirling number |  |
| 1.1 $\quad$ Four basic counting principles: Addition principle, multiplication principle, subtraction principle, division principle Permutation of a set and Combination of a set | [1L] |
| 1.2 Counting sets of pairs, two-way counting Circular Permutation, Permutation of a multi set, combination of a multi set. | [2L] |



## References

- Richard A. Brualdi; Introductory Combinatorics; Pearson (4th Edition)
- Alan Tucker; Applied Combinatorics; John Wiley and sons (6th edition)
- V. Krishnamurthy: Combinatorics - Theory and Applications, Affiliated East West Press.


## Module 3 <br> Learning Objectives

Generating functions and recurrence relation

## [12L]

The learner should be able to

1. Understand the purpose of generating functions
2. Study generating functions
3. Evaluate the coefficients of a generating function
4. Comprehend various methods to solve homogeneous and non-homogeneous recurrence relations

## Learning Outcomes

At the end of the module, the learner will be able to

1. Calculate their coefficients of generating functions
2. Identify the appropriate method to obtain the generating function
3. Solve homogeneous and non-homogeneous recurrence relations

| 3.1 | Review : Arithmetic and Geometric sequence <br> problems of four mutually overlapping circles, Fibonacci sequence. <br> Relationship of binomial coefficient along the diagonals of a Pascal's <br> triangle with Fibonacci sequence | [3L] |
| :--- | :--- | :--- |
| $\mathbf{3 . 2}$ | Generating Function Models, Calculating Coefficients of Generating <br> Functions <br> Partitions, Ferrers diagram <br> Exponential Generating Functions | [5L] |
| $\mathbf{3 . 3}$ | Linear Homogeneous Recurrence relations. <br> Non-Homogeneous Recurrence Relations. <br> Recurrences and Generating functions. | [4L] |

## References

- Alan Tucker; Applied Combinatorics; John Wiley and sons (6th edition)
- Richard A. Brualdi; Introductory Combinatorics; Pearson (4th Edition)
- V. Krishnamurthy: Combinatorics - Theory and Applications, Affiliated East West Press.

T.Y. B. Sc. (Mathematics) SEMESTER VI<br>Discipline Specific Course- IV<br>COURSE TITLE: Game Theory<br>COURSE CODE: 23US6MTDS4GMT CREDITS - 02]

## Learning outcome:

After the successful completion of the Course, the learner will be able to:
CLO 1. Understand strategies to analyze games to solve the problem
CLO 2. Explore principle of dominance
CLO 3. Solve zero sum games

| Module I Analysing games |
| :--- |
| Objective: Aim of this module is to introduce Game theory, its Characteristics, And zero-sum |
| game |
| Learning Outcome: A student after learning this module is expected to solve simple competitive |
| Games graphically. |
| Competitive games, Characteristic of competitive games, |
| zero-sum and non-zero-sum games, Two-persons Zero-sum games, conservative players, saddle |
| point and value of a game. |
| Maximin-Minimax criterion, Games without saddle points, |
| Graphic method for 2xn and mx2 Games. |
| Combinatorial games |
| General-sum games, Nash equilibria |
| Module II Principle of Dominance |
| Objective: Understand the Principles of solving Zero-sum games. |
| Learning outcomes: A student after learning this course is expected to understand the principle |
| behind zero-sum games and how one can solve them. |
| Symmetric game, Minimax and saddle point theorem, |
| Fundamental theorem of matrix Game, |
| Principle of Dominance |
| Module III Methods to solve Zero-sum games |
| Objective: Aim of this module is to introduce some of the methods that can be employed to |
| solve a zero-sum problem. |
| Learning outcome: After learning this module students are expected to know how to solve some |

## of the zero-sum problems.

Connection between Game and LP. Algebraic method for $\mathrm{m} \times \mathrm{n}$ Games, Iterative method for approximate solution, Extension of two person games.

References Sultan chand and sons

1. Linear Programming and theory of games, P.K. Gupta and Manmohan
2. Game Theory, Thomas S. Ferguson.
3. Essentials of Game Theory_Kevin Leyton-Brown and Yoav Shoham.
4. gametheory.net
5. (IGT) Martin Osborne, An Introduction to Game Theory, Oxford University Press, 2003
6. (AT) Vijay Krishna, Auction Theory, Academic Press.
7. (SG) PrajitDutta, Strategies and Games, MIT Press
8. (Website 1) http://www.ece.stevens-tech.edu/~ccomanic/ee800c.html
9. (GTWE) Allan MacKenzie, Game Theory for Wireless Engineers, Synthesis lectures on Communications, 2006
10. (IITD Website)
11. (HV) Hal Varian, Microeconomic Analysis, Norton
12. (Gandhi) Gandhi et.al.,Towards Real-Time Dynamic Spectrum Auctions by Gandhi
13. Game Theory, Alive._Anna R. Karlin and Yuval Peres. 園 (Sep 25, 2016 version)

## T.Y. B. Sc. (Mathematics) SEMESTER VI <br> Discipline Specific Course- V <br> COURSE TITLE: Mathematical elements in computer graphics COURSE CODE: 23US6MTDS5ECG CREDITS - 02

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Explore two dimensional transformations
CLO 2: Understand three dimensional transformations
CLO 3: Understand plane and space curves

## Module 1 Two dimensional Transformations

## Learning Objectives:

The learner should be able to:
understand two dimensional transformations

| Learning Outcome: <br> After the successful completion of the module, the learner will be able to: <br> understand two dimensional transformations |  |  |
| :--- | :--- | :--- |
| 1.1 | Introduction, Representation of Points, Transformations and Matrices,. | [1L] |
| 1.2 | Transformation of Points, Transformation of Straight Lines, Midpoint | [2L] |
| 1.3 | Transformation, Transformation of Parallel Lines, Transformation of <br> Intersecting Lines, Rotation, Reflection, Scaling, Combined <br> Transformations | [3L] |
| 1.4 | Transformation of <br> the Unit Square, Solid Body Transformation, Translations and <br> Homogeneous <br> Coordinates, Rotation About an Arbitrary Point, Reflection Through an <br> Arbitrary <br> Line. | [2L] <br> Orojection - A Geometric Interpretation of Homogeneous Coordinates, <br> Scaling, Points at Infinity. |
| [2L] |  |  |
| Reference books |  |  |
| - | D.F. Rogers, J. Alan Adams, Mathematical Elements of Computer Graphics, Second <br> Edition, McGraw-Hill Publishing Company. |  |
| - John Vince, Matrix transforms for computer games and animations. |  |  |
| - John Vince, Mathematics for Computer Graphics. |  |  |



- D.F. Rogers, J. Alan Adams, Mathematical Elements of Computer Graphics, Second Edition, McGraw-Hill Publishing Company.
- John Vince, Matrix transforms for computer games and animations.
- John Vince, Mathematics for Computer Graphics.
- Steven J Jenke, Mathematical structures for computer graphics.


## Learning Objectives:

The Practical is intended to

1. Solve problems based on the concepts learnt
2. Apply the concepts in various situation

## Learning Outcome:

After the successful completion of the practical, the learner will be able to:

1. Solve problems
2. Apply the results proved
3. Generate examples and counterexamples
1.1 Problems based on finite continued fraction, solving linear Diophantine equation using Continued Fraction, $\mathrm{K}^{\text {th }}$ convergent $\left(\mathrm{C}_{\mathrm{k}}\right)$ of a continued fraction.
1.2 Problems based on Representation of an irrational number as an infinite simple continued fraction, Rational approximation of an irrational number.
1.3 Problems based on Solutions of Pell's equation, Arithmetic functions of number theory: $d(n)$ ( $\operatorname{or} \mathrm{t}(\mathrm{n})$ ), $\sigma(\mathrm{n})$ and their properties. $\mu(\mathrm{n})$ and the Mobius inversion formula.
1.4 Problems based on Special numbers: Fermats numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers. Algebraic and transcendental numbers. Algebraic integers, minimal polynomial of algebraic numbers.
1.5 Problems based on Classical Cryptosystems, Shift cipher, affine cipher, Vigenere cipher, Play Fair cipher, ADFGX cipher.
1.6 Problems based on Block Cipher, Public Key cryptosystem, RSA algorithm

Elective 2
Java programming

Learning Objectives:
The Practical is intended to

1. Write programs implementing Mathematical concepts
2. Creating simple animations

## Learning Outcome:

After the successful completion of the practical, the learner will be able to:

1. Write simple and complex programs
2. Implementing Mathematical concepts using Java
3. Create applets leading to simple animation

| 1.1 one class and two class programs illustrating arrays and use of objects |  |
| :---: | :---: |
| 1.2 two class programs illustrating default constructor, constructors with parameters and the keyword this. |  |
| 1.3 programs illustrating inheritance |  |
| 1.4 Programs illustrating overloading and overriding problems |  |
| 1.5 Working with applets for creating geometrical figures which can lead to simple animation. |  |
| 1.6 Applets with setting fonts and different background, Strings within Geometrical figures |  |
| Elective 3 | Combinatorics |
| Learning Objectives: <br> The Practical is intended to <br> 1. Solve problems based on the concepts learnt <br> 2. Apply the concepts in various situation |  |
| Learning Outcome: <br> After the successful completion of the practical, the learner will be able to: <br> 1. Solve problems <br> 2. Apply the results proved |  |
| 1.1 Problems based on permutations, combinations, two-way counting |  |
| 1.2 Proving equalities/inequalities using combinatorial reasoning and problems based on binomial and multinomial theorem. |  |
| 1.3 Problems based on principles of inclusion and exclusion and derrangements. |  |
| 1.4 Problems based on pigeon hole principle |  |
| 1.5 Problems based on generating functions |  |
| 1.6 Problems to solve homogeneous and non-homogeneous recurrence relations |  |
| Elective 4 | Game theory |


| Learning Objectives: <br> The Practical is intended to <br> 1. Solve problems based on the concepts learnt <br> 2. Apply the concepts in various situation |
| :--- |
| Learning Outcome: <br> After the successful completion of the practical, the learner will be able to: <br> 1. Solve problems <br> 2. Apply the results proved <br> 1.1 Problems based on Zero-sum problems <br> 1.2 Problems based on Non-Zero-sum problems <br> 1.3 Problems based on Symmetric games, minimax and saddle-point <br> 1.4 Problems based on dominance principle <br> 1.5 Problems based on mxn games <br> 1.6 Problems based on extension of two person games <br> Elective 5 <br> Learning Objectives: <br> The Practical is intended to <br> Learning Outcome: <br> 2. Apply the concepts in various situation <br> After the successful completion of the practical, the learner will be able to: <br> 1. Solve problems <br> 2. Apply the results proved <br> 1.1 Problems based on Scaling, shearing, rotations, reflections and translations of 3 dimensional |

1.4 Orthographic, axonometric projections, perspective transformations
1.5 parametric and non-parametric representations of plane curves and space curves
1.6 Bezier curves

## Students are required to select any one courses

Of the following Generic courses, learners can select any one of the courses. The courses having maximum number of votes will be considered for the year and subject to availability of the concerned teacher.

| DSE | Title |
| :--- | :--- |
| GE 1 | Operation Research - II |
| GE 2 | PL/SQL |

## T.Y. B. Sc. (Mathematics) SEMESTER VI <br> General Elective - I <br> COURSE TITLE: Operations Research-II <br> COURSE CODE: 23US6MTGEOR CREDITS - 02]

This is a practical oriented course

After the successful completion of the Course, the learner will be able to:
CLO 1: Solve transportation problem
CLO 2: Solve assignment problem

## Module 1 Transportation problem

## Learning Objectives:

The learner should be able to:
Find Initial Feasible initial basic solution by the prescribed methods
Solve transportations
Problem and allied problems using MODI method
Learning Outcome:
After the successful completion of the module, the learner will be able to:

1. Find initial basic feasible solution by North-west corner, Matrix minimum and VOGLEs method
2. Solve transportation problem and its variations with restrictions using MODI method

| 1.1 | Transportation problem (TP) and its formulation. Finding initial basic feasible <br> solution of TP using North-West Corner Rule, Least Cost and Vogel's <br> Approximation Method, problem. | [10L] |
| :--- | :--- | :--- |
| 1.2 | MODI method for finding optimal solution for TP, | [10L] |

Reference books

- G. Hadley: Linear Programming. Narosa, Reprint, 2002.
- Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, 2010.
- A. Ravindran, D. T. Phillips and James J. Solberg: Operations Research- Principles
and Practice, John Wiley \& Sons, 2005.
- F.S. Hillier. G.J. Lieberman: Introduction to Operations Research- Concepts and Cases, 9th Edition, Tata Mc-Graw Hill, 2010.


## Module 2 Assignment Problem

## Learning Objectives

The learner should be able to:
4. Understand various concepts used in Linear Programming
5. Use Graphical methods to solve Linear Programming problems
6. Use Iso cost line in graphical solutions

## Learning outcomes

The learner will be able to:
3. Formulate the linear programming problems
4. Solve the linear programming problems graphically

| 2.1 | Assignment problem and its formulation, | [6L] |
| :--- | :--- | :--- |
| 2.2 | Hungarian method for solving Assignment problem, Transhipment and <br> Travelling salesmen | [10L] |

Reference books

- G. Hadley: Linear Programming. Narosa, Reprint, 2002.
- Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, 2010.
- A. Ravindran, D. T. Phillips and James J. Solberg: Operations Research- Principles and Practice, John Wiley \& Sons, 2005.
- F.S. Hillier. G.J. Lieberman: Introduction to Operations Research- Concepts and Cases, 9th Edition, Tata Mc-Graw Hill, 2010.


## T.Y. B. Sc. (Mathematics) SEMESTER VI <br> General Elective - II <br> COURSE TITLE: PL/SQL <br> COURSE CODE: 23US6MTGEPLSQL CREDITS - 02]

This is a practical oriented course

After the successful completion of the Course, the learner will be able to:
CLO 1: Write simple programs

| Module 1 | PL/SQL | [20L] |
| :--- | :--- | :--- |

Learning Objectives:
The learner should be able to:
Understand command and operators
Write simple programs
Learning Outcome:
After the successful completion of the module, the learner will be able to:

1. Understand how to write simple programs

| 1.1 | Features and advantages, Identifiers, Delimiters, commands, Data types <br> and subtypes, date time, NULL | [10L] |
| :--- | :--- | :--- |
| 1.2 | Variables, Literals, arithmetic operators, relational operators, logical <br> operators, LIKE operators, BETWEEN operators, IN and ISNULL operators, <br> conditional statement, loops, labeling a loop, EXIT WHEN statement | [26L] |
| Reference books <br> - PL/SQL Tutorial |  |  |

T.Y.B.Sc. (Mathematics) Semester VI

## Skill Enhancement courses

## T.Y. B. Sc. (Mathematics) SEMESTER VI Skill Enhancement Course COURSE TITLE: Wx-Maxima COURSE CODE: 23US6MTSE1MAX CREDITS - 02

## Course Learning Outcome

After the successful completion of the Course, the learner will be able to:
CLO 1: Solve problems in linear algebra, calculus and number theory using Wx -maxima
CLO 2: Plot 2-D and 3-D graphs using Wx-maxima
CLO 3: $\quad$ Solve first order linear differential equation using Wx-maxima

| Module 1 | Linear Algebra, Calculus | [6L] |
| :--- | :--- | :--- |

## Learning Objectives

The learner should be able to:

1. Use the syntax and process to solve mathematical problems

## Learning Outcomes

After the successful completion of the module, the learner will be able to:

1. Use Wx-maxima software in solving problems of linear algebra and calculus

| 1.1 | Matrices, rank, characteristic polynomial, eigen values, eigen vectors | [2L] |
| :--- | :--- | :--- |
| 1.2 | System of equations, Roots of polynomial | [2L] |
| 1.3 | Derivatives, integration, limit, series | [2L] |

Reference books

1. Wx-maxima Manual

Module 2 Graphs, Ordinary differential equations, number theory

## Learning Objectives

The learner should be able to:

1. Use the syntax and process to solve mathematical problems

## Learning outcomes

The learner will be able to:

| 1. Use Wx-maxima software in plotting 2D and 3D graphs <br> 2. Solve first order differential equations using Wx-maxima <br> 3. Find continued fraction of any real number |  |  |
| :--- | :--- | :--- |
| 2.1 | 2D and 3D graphs | [3L] |
| 2.2 | First order differential equations | [2L] |
| 2.3 | Continued Fraction | [1L] |
| Reference books <br> 1. Wx-maxima manual |  |  |

## Guidelines about conduct of Projects/Case Study.

## - Projects/ Case Study/ Book Review:

Conduct and Evaluation: A learner can submit a project/ Case Study/ do a Book review. The project should be 10-page typed pages in an A4 size paper with font size of 12. The topic of project should be selected in consultation of the teacher. Maximum marks allotted for this is $\mathbf{2 0}$ and the remaining $\mathbf{2 0}$ marks are from tests and other activities.

The topic can be of expository / historical survey / interdisciplinary nature and the material covered in the project / case study should go beyond the scope of the syllabus. The learner must clearly mention the sources (Book / on-line) used for the project/ case study. The use of Mathematical software is encouraged. The project should be done under the supervision of a faculty in a college/ Institution / University.

The following Marking scheme is suggested for evaluation of projects / case study:
$30 \%$ marks for exposition
20\% marks for literature
20\% marks for Scope
10\% marks for originality
20\% marks for presentation.

## Continuous evaluation:

## Internal evaluation (40\%):

1. There will be 40 marks continuous evaluation.
2. A learner can be assigned projects/book review, this will be evaluated out of 20 marks.
3. The project / book review will be under the guidance of the mentor allotted to the learners by the head of the department.

- There will be regular tests which can be of the form quiz/ descriptive test/ objective test/ group discussion presentation etc.
- Each test will be marked out of 20 marks.
- The total score obtained in all of the above will finally be averaged to 40 marks.
- A learner should secure at least $40 \%$ marks to be eligible to get a passing grade (The learner needs to secure minimum of 16 marks out of 40 to pass the internal for each theory course).
- A learner who has failed to secure a passing grade /absent for any reason in the internal evaluation will have to give test out of 40 marks, consisting of Questions based on the entire syllabus.


## Semester end Examination (60\%):

At the end of the semester there will be a semester end exam carrying a maximum of 60 marks.

1. There will be 4 Questions one from each Module. Each question will carry 15 marks unless otherwise stated in the syllabus (with option, maximum of 25 marks). The question paper will cover the whole syllabus in such a way that a learner will need to have understood each topic well to have secured $80 \%$ and above and an average learner can at least secure a passing grade.

- A learner should secure at least $40 \%$ marks to be eligible to get a passing grade (The learner needs to secure minimum of 24 marks out of 60 to pass the semester end examination for each theory course).


## Practical examination

2. Practical Examination out of 100 marks will be conducted based on the theory courses.
3. $40 \%$ evaluation will be based on continuous evaluation and balance $60 \%$ will be Semester end examination.
4. Certified Journal will be part of internal evaluation.
5. Internal evaluation will be based on experiential learning such as preparing Mathematical model/ Games/quizzes, Applying Concepts learnt in other areas of mathematics or other Sciences, Presentations.
6. Contribution during Cooperative/Participative learning will be evaluated during regular practical. No prior intimation will be given.
7. Semester end examination of the Practical examination will be descriptive and will be based on the entire syllabus of both theory courses.
distribution of marks for practical examination out of 100. (corresponding modification for exam conducted out of 150 marks)

Mathematical subject/Generic courses

|  | Course 1 | Course 2 | Total |
| :---: | :---: | :---: | :---: |
|  | internal continuous <br> assessment | Generic <br> courses |  |
| objective questions | 6 | 6 | 12 |


| journal | 5 | 5 | 10 |
| :---: | :---: | :---: | :---: |
| viva | 5 | 5 | 10 |
| Modelling | 4 | 4 | 8 |
| Total | 20 | 20 | 40 |
|  |  |  |  |
|  |  |  |  |
|  | Semester end descriptive problem solving |  |  |
| Comprehension kind |  | 6 | 12 |
| Application type | 8 | 8 | 16 |
| Analysis type | 8 | 8 | 16 |
| evaluation/creating type | 8 | 8 | 16 |
| Total | 30 | 30 | 60 |


| For Computer programming courses |  |  |
| :---: | :---: | :---: |
|  | internal continuous assessment |  |
|  | project | 20 |
| making modifications/writing the required statements as per constraints given/new. (includes viva) |  | 15 |
|  | Total | 40 |
| Semester end practical examination |  | Marks |
| Writing programme (2 programmes) | applying it to mathematical concepts learnt (creating kind) | 15 |
|  | understanding type Applying type | 10 |
|  | compiling and execution | 5 |
|  | correcting errors and obtaining output | 10 |
|  | Viva and journal | 10 |
|  | Total | 40 |

## Examination for unsuccessful learners (Termed as ATKT examination)

- Internal examination will be a test conducted out of 40 marks based on the entire syllabus. It will be written test/ online test as per the situation. Details of the pattern etc will be uploaded in the noticeboard section of our website kjssc.somaiya.edu
- Semester Exam will have the same paper pattern as the regular exam. (Subject to change.)
- Internal Component of the Practical Examination (40\%) will be objective based examination. This will include journal marks (only Certified Journal will be eligible for marks)
- Notice regarding syllabus will be uploaded in the noticeboard section in our website.

