SOMAIYA VIDYAVIHAR
K J Somaiya College of Science & Commerce
Autonomous (Affiliated to University of Mumbai)

Somaiya TRUST

Learning Outcomes based Curriculum Framework (LOCF) For

F.Y.B.Sc.
Mathematics (Autonomous)

Undergraduate

Under NEP Programme Guidelines From

Academic year 2023-24

F.Y. B. Sc. (Mathematics) SEMESTER I
Core Course- I
COURSE TITLE: Calculus I
COURSE CODE: 23US1MTCC1CAL1 CREDITS - 02

Course Learning Outcomes	
After the successful completion of the Course, the learner will be able to:	
CLO 1:	Analyse convergence of sequence and its subsequence.
CLO 2:	Prove results on Cauchy sequences in R.
CLO 3:	Test convergence of series using different tests.
CLO 4:	Apply results proved in mathematical sciences and real world modelling

Module 1	Sequence and Subsequence of Real numbers	[15L]

Learning Objectives:
The learner should be able to:

1. Understand the meaning of a sequence
2. Understand the concept of limit of a sequence
3. Apply the LUB property to monotonic bounded sequences
4. Obtain e, through a sequence
5. Understand the fact that for a convergent sequence, all subsequences give the same limit

Learning Outcomes:
After the successful completion of the module, the learner will be able to:

1. Use of sequential approach for solving problems

	2. Apply tools developed in module I for proving results and solving problems 3. Solve problems based on subsequences.	
1.1	ε-neighbourhood of a point in $\mathbb{R}$. Upper bound and lower bound of a subset of $\mathbb{R}$. Supremum (lub) and infimum (glb) of a non-empty subset of $\mathbb{R}$. Characterization of lub and glb in terms of ε. Archimedean property of real numbers.	[3L]
1.2	Sequence of real numbers, $\varepsilon - n_0$ definition, limit of a sequence, Uniqueness of Limit. Bounded Sequence, Convergent sequence is bounded. Algebra of convergent sequences (Self-Study). The number e as a limit of a sequence. Monotonic sequences, every monotone bounded sequence is convergent. Examples of sequence of rational numbers converging to $\sqrt{a},\ a \in R$. Standard examples such as $a^n,\ a^{1/n},\ n^{1/n},\ (n!)^{1/n}$. Statement of Density Theorem (Results and problems part of project), Every real number can be expressed as a limit of a sequence of rational numbers and also as limit of sequence of irrational numbers.	[8L]
1.3	Subsequence of a sequence. Convergence of a sequence implies convergence of its subsequence but not conversely. Convergence of (x_{2n}), (x_{2n-1}) to the same limit p implies convergence of (x_n) to p. Every bounded sequence has a convergent subsequence.	[4L]

Reference Books:

- R.G. Bartle and D. R Sherbert; Introduction to Real Analysis; John Wiley and Sons (Asia) P.Ltd.
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- R. R. Goldberg; Methods of Real Analysis; Oxford and IBH.
- Ajit Kumar, S. Kumaresan; A Basic Course in Real Analysis; CRC Press.
- Peter D Lax, Maria Shea Terrel; Calculus with applications, Springer

Additional Reference books:

- H. Anton, I. Bivens and S. Davis; Calculus; John Wiley and Sons, Inc.
- Ghorpade, Sudhir R., Limaye, Balmohan V.; A Course in Calculus and Real

Analysis; Springer.
- T. M. Apostol; Calculus (Vol. I); John Wiley and Sons (Asia) P. Ltd.
- Maron; Calculus of one variable Arihant
- Shanti Narayan and Raisinghania; Elements of Real Analysis; S. Chand

Module 2	Cauchy Sequence and Series of real numbers	[15L]

Learning Objectives

The learner should be able to:

1. Prove "Cauchy's criteria for convergence of sequence
2. Understand that a sequence can be written as a series and vice versa
3. Prove the summability of series in terms of sequences of partial sums for simple examples
4. Recognize the appropriate tests which can be applied for particular problems
5. Appreciate the importance of Geometric series and p-series in solving problems on comparison test
6. Understand that these tests are not exhaustive

Learning outcomes

The learner will be able to:

1. Determine whether the sequence is Cauchy
2. Analyse the given problem on convergence of series with appropriate choice of test

2.1	Definition of a Cauchy sequence. Every convergent sequence is Cauchy. Every Cauchy sequence is bounded. If a subsequence of a Cauchy sequence is convergent then the sequence itself is convergent. Every Cauchy sequence of real numbers is convergent. Completeness of R.	**[6L]**
2.2	Series of real numbers. Terms of a series and partial sums. Summability / Convergence of a real series in terms of convergence of its partial sums. Convergence of series implies convergence of n^{th} term to zero, and converse is false. Simple examples of convergent series and divergent series without involving tests. Sum of two convergent series is convergent. If every term of a convergent series is multiplied by a constant, then the resultant series is convergent. Term wise product of two convergent series need not result into a	**[9L]**

	convergent series. Geometric series; it converges if and only if the common ratio lies in $(-1, 1)$. Series of nonnegative terms. Cauchy's condensation test. p-series converges if and only if $p > 1$. Comparison test in simple form. Alternating series and Leibnitz' test. Absolute convergence and conditional convergence. Absolute convergence implies conditional convergence and converse is false.Ratio test and root test (Statement only) and problems based on these tests.	

Reference Books:

- R.G. Bartle and D. R Sherbert; Introduction to Real Analysis; John Wiley and Sons (Asia) P.Ltd.
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- R. R. Goldberg; Methods of Real Analysis; Oxford and IBH.
- Ajit Kumar, S. Kumaresan; A Basic Course in Real Analysis; CRC Press.
- Peter D Lax, Maria Shea Terrel; Calculus with applications, Springer

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- H. Anton, I. Bivens and S. Davis; Calculus; John Wiley and Sons, Inc.
- Ghorpade, Sudhir R., Limaye, Balmohan V.; A Course in Calculus and Real Analysis; Springer.
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F.Y. B. Sc. (Mathematics) SEMESTER I
Core Course- II
COURSE TITLE: Algebra I
COURSE CODE: 23US1MTCC2ALG1 CREDITS - 02

Course Learning Outcomes	
After the successful completion of the Course, the learner will be able to:	
CLO 1:	Apply the properties of natural numbers, integers and prime numbers
CLO 2:	Solve problems on linear Diophantine equations.
CLO 3:	Analyse various algebraic properties of polynomials and find roots using various tools.
CLO 4:	Comprehend complex numbers; its polar representation and algebraic properties and appreciate geometric interpretation through related problems

Module 1	Integers and Diophantine equations	[15L]

Learning Objectives:
The learner should be able to:

1. Develop the properties of integers and prime numbers
2. Solve problems on linear Diophantine equations

Learning Outcomes:
After the successful completion of the module, the learner will be able to:

1. Apply the knowledge of integers and prime numbers

2. Solve linear Diophantine equations		
1.1	Statement of well-ordering principle, Principle of finite induction (first and second) as a consequence of well-ordering property	[2l]
1.2	Binomial theorem for non-negative exponents, Pascal Triangle, Multinomial Theorem	[3L]
1.3	Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a & b and that the g.c.d. can be expressed as ma +nb for some m,n $\in$ **Z**, Euclidean algorithm	[4L]
1.4	Primes, Euclid's lemma, Fundamental theorem of arithmetic, The set of primes is infinite	[2L]
1.5	Linear Diophantine equation $ax + by = c$ The linear Diophantine equation $ax + by = c$ has solution if and only if $d \mid c$, where $d = GCD(a, b)$. If x_0, y_0 is any particular solution then any solution of the given Diophantine equation is given by $x = x_0 + (\frac{b}{d})t$ and $y = y_0 - (\frac{a}{d})t$, for varying t. Solving examples.	[4]

Reference Books:

- An Introduction to the Theory of Numbers by G. H. Hardy and E. M. Wright, fourth edition, Oxford at the Clarendon Press
- Elementary Number theory by David Burton Seventh Edition, McGraw Hill Education (India) Pvt Ltd.

Additional Reference books::

- Niven and S. Zuckerman; Introduction to theory of numbers; John Wiley & Sons, Inc.

Module 2	Polynomials and Complex Numbers	[15L]

Learning Objectives

The learner should be able to:

1. Analyse various algebraic properties of polynomials and find roots using various tools
2. Appreciate geometric interpretation of complex numbers through related problems

Learning outcomes

The learner will be able to:

1. Obtain roots of polynomials with real, rational or integer coefficients using various techniques
2. Compute complex roots of unity, and roots of any complex number in general

2.1	Definition of a polynomial, polynomials over **R**, Algebra of polynomials, degree of polynomial, basic properties, Division algorithm in **R**[X] (without proof), and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications	[4L]
2.2	Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem, A polynomial of degree n has at most n roots, Rational Root Theorem, Relation between roots and coefficients of a polynomial	[5L]
2.3	Review of Complex numbers, Complex roots of a polynomial in **R**[X] occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A non-constant polynomial in **R**[X] can be expressed as a product of linear and quadratic factors in **R**[X]	[3L]
2.4	De Moivre's Theorem, roots of unity, sum of all the roots of unity	[3]

Reference books

- Titu Andreescu, Dorin Andrica; Complex numbers from A to Z; Birkhauser
- Michael Artin; Algebra; Birkhauser(Section 11.2)

Additional Reference books:

- G. Birkhoff and S. Maclane; A survey of Modern Algebra; AMS Chelsea Publication
- Brown and Churchill; Complex variables and Applications; McGraw Hill.

F. Y. B. Sc. (Mathematics)
SEMESTER I - Practicals
COURSE CODE: 23USIMTCCP Credits- 02

Learning Objectives:

The Practical is intended to

1. Solve problems based on the concepts learnt
2. Apply the concepts in various situation

Learning Outcome:

After the successful completion of the practical, the learner will be able to:

1. Apply the results proved to the other sciences
2. Create examples and counterexamples
3. Solve modern and classical problems

Module I	Calculus 1
1.1 Problems based on ε-neighbourhood, real numbers, Supremum, Infimum, Archimedean property	
1.2 Problems on Sequence and Subsequence of real numbers	
1.3 Problems on Cauchy Sequence	
1.4 Series I	
1.5 Series II	
1.6 Formal/Mathematical proof writing such as : a. Monotone Convergence Theorem b. Monotone Subsequence Theorem and Bolzano-Weierstrass Theorem c. p-series convergence d. Leibniz test	

Module II	Algebra-1
1.1 Induction, Binomial Theorem, Multinomial Theorem	

1.2 G.C.D, L.C.M, Primes
1.3 Diophantine equations
1.4 Roots of Polynomials
1.5 Complex numbers
1.6 Mathematical writing/ Formal Proof writing such as Binomial Theorem and Euclid's Lemma
1.7 Mathematical writing/ Formal Proof writing such as Rational Root Theorem and De' Moivre's Theorem

F.Y. B. Sc. (Mathematics) SEMESTER II
Core Course- I
COURSE TITLE: Calculus II
COURSE CODE: 23US2MTCC1CAL2 CREDITS - 02

Course Learning Outcomes		
After the successful completion of the Course, the learner will be able to: CLO 1: Prove various results of limits and continuity using the $\varepsilon - \delta$ definition and sequential criterion. CLO 2: Prove Chain rule, inverse function theorem, Leibnitz theorem and Mean value theorem. CLO 3 Apply Taylor's theorem to generate power series expansion of standard functions. CLO 4: Apply results proved, concepts defined to solve problems.		
Module 1	**Limits and continuity of real valued functions of one variable**	**[15L]**
Learning Objectives: The learner should be able to: 1. Understand the $\varepsilon - \delta$ definition of limit and continuity 2. Comprehend the equivalence of $\varepsilon - \delta$ definition of continuity with the sequential continuity 3. prove algebra of limits and continuity 4. Obtain limit of a function at a point		
Learning Outcomes: After the successful completion of the module, the learner will be able to:		

1. Use $\varepsilon - \delta$ definition as well definition in terms of sequences to find limit of a function 2. Use $\varepsilon - \delta$ definition of continuity as well as sequential continuity to prove the continuity or discontinuity of a function at a given point 3. Find the left-hand and the right-hand limit of a function 4. Construct functions having discontinuity at desired points		
1.1	Definition of limit of a function at a point in terms of $\varepsilon - \delta$. Uniqueness of limit. Boundedness of a function having limit in a neighbourhood. Concept of two sided and one-sided limits. Sum rule, scalar multiplication, product rule and division rule. Sandwich theorem. Computations of limits using rules. Nonexistence of limit of functions such as sin $sin\ \frac{1}{x}$. Infinite limit and limit at infinity.	[7L]
1.2	Definition of continuity of a function at a point in terms of ε - δ and sequence. Equivalence of both the definitions. Continuity of a function at a point in terms of limits. Continuity of a function over an interval, over a set. Algebra of continuous functions. Discontinuity of function such as $si(1/x)$ at the origin, step function etc. Polynomial functions are continuous. Function such as $\|x\|$, $x\|x\|$, $xsin(1/x)$ etc are continuous. Removable and irremovable discontinuities. Functions having finite number of discontinuities in an interval. Functions having infinite number of discontinuities in an interval. A function which is discontinuous everywhere. A function which is continuous only at one point. Composition of continuous functions and taking a limit inside a continuous function. Composite of continuous functions is continuous but converse is not true. Two important properties of Continuous functions: Intermediate value property and Continuous function on a closed and bounded interval attains its maximum value and minimum value. (Without proof)	[8L]
Reference Books: • R.G. Bartle and D. R Sherbert; Introduction to Real Analysis; John Wiley and Sons (Asia) P.Ltd.		

- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
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- Shanti Narayan and Raisinghania; Elements of Real Analysis; S. Chand

Module 2	Differentiability of real valued functions of one variable and its application	[15L]

Learning Objectives

The learner should be able to:

1. Analyse geometrical interpretation of derivative
2. Determine Higher order derivatives of a function
3. Obtain Taylor's polynomial and Taylor's series of a function about a point
4. Apply the concept of differentiation to solve real world problems.

Learning outcomes

The learner will be able to:

1. Analyse differentiability of a function
2. Apply results proved to solve mathematical science and real world problems
3. Create examples and counterexamples

2.1	Definition of differentiability of a real valued function of one variable at a point in terms of a limit. Differentiability of a function over an interval or a set. Geometrical interpretation of derivative. Derivative as rate of change and Leibnitz notation. Differentiability implies continuity but not converse. Algebra of derivatives. Chain rule of differentiation. Inverse function theorem. Implicit differentiation. Rolle's Mean Value Theorem, Lagrange's Mean Value Theorem. Higher order derivatives and Leibnitz' rule. Function that are only n times differentiable.	**[9L]**

2.2	Taylor's theorem in Lagrange's remainder form . Taylor polynomial of n^{th} order and Taylor's series about a point. Approximation of a n times differentiable function using Taylor's theorem. Increasing and decreasing functions. Concavity in terms of second derivative. Point of inflection. Local extreme values. Second derivative test and its extension to test using higher order derivatives. Applications of L'Hospital's Rule	**[6L]**

Reference Books:

- R.G. Bartle and D. R Sherbert; Introduction to Real Analysis; John Wiley and Sons (Asia) P.Ltd.
- G.B. Thomas and R.L. Finney; Calculus; Pearson Education.
- R. R. Goldberg; Methods of Real Analysis; Oxford and IBH.
- Ajit Kumar, S. Kumaresan; A Basic Course in Real Analysis; CRC Press.
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F.Y. B. Sc. (Mathematics) SEMESTER II
Core Course- II
COURSE TITLE: Linear Algebra I
COURSE CODE: 23US2MTCC2LALG1 CREDITS - 02

Course Learning Outcomes	
After the successful completion of the Course, the learner will be able to:	
CLO 1:	Solve problems based on various types of Matrices and their properties.
CLO 2:	Apply Gauss Elimination method to solve a system of linear equations
CLO 3	Verify properties of a vector space and subspaces
CLO 4:	Compute basis and dimension

Module 1	Matrices	[15L]

Learning Objectives:
The learner should be able to:

1. Learn the various types of matrices, operations on matrices, decomposition,eigen values and eigen vectors
2. Apply the methods to solve systems of linear equations with the geometric interpretation, followed by their simple applications

Learning Outcomes:
After the successful completion of the module, the learner will be able to:

1. Solve problems on decomposition,eigen values and eigen vectors
2. Solve problems based on system of linear equations

1.1	Matrices with real entries; addition, scalar multiplication and multiplication of matrices; transpose of a matrix, types of matrices: zero matrix, identity matrix, scalar matrices, diagonal matrices, upper triangular matrices, lower triangular matrices, symmetric matrices, skew-symmetric matrices, Invertible matrices; identities such as $(AB)^t = B^t A^t$, $(AB)^{-1} = B^{-1}A^{-1}$. System of linear equations in matrix form, elementary row operations, row echelon matrix, Orthogonal matrix, Hermitian matrix, block multiplication	[5L]
1.2	LU decomposition, QR decomposition, singular value decompositions, convolutions, two and three dimensional rotations	[3L]
1.3	eigen values and eigen vectors of a matrix (proofs of simple properties), examples	[2L]
1.4	System of homogeneous and nonhomogeneous linear equations, the solution of system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation	[3L]
1.5	Gaussian elimination method, to deduce that the system of m homogeneous linear equations in n unknowns has a non-trivial solution if m < n	[2]

Reference books

- S. Kumaresan; Linear algebra -A geometric approach; Prentice Hall of India(2000)
- Gilbert Strang; Linear Algebra; Wellesy-Cambridge Press
- M.K.Jain, S.R.K Iyengar, R.K.Jain; Numerical Methods(Problems and Solutions); New Age International (P) Limited, Publishers

Additional Reference books

- K Hoffman and R Kunze ; Linear algebra; Prentice-Hall
- Sheldon Axler; Linear Algebra Done Right; Springer

Module 2	Vector Spaces	[15L]

Learning Objectives

The learner should be able to:

1. Comprehend the concept of vector space and subspace
2. Test the linear independence and generating property leading to the study of basis and dimension

Learning outcomes

The learner will be able to:

1. Solve problems related to various concepts in vector spaces using definitions and properties
2. Determine basis, dimension and rank of a matrix

2.1	Definition of a real vector space, examples such as $\mathbf{R}^n$, $\mathbf{R}[X]$, $M_{m\times n}[\mathbf{R}]$, space of all real valued functions on a non-empty set(e.g. [a,b]), Finite linear combinations of vectors in a vector space; the linear span L(S) of a non-empty subset S of a vector space	**[5L]**
2.2	Subspace: definition, examples: lines, planes passing through origin as subspaces of $\mathbf{R}^2$,$\mathbf{R}^3$ respectively; Subspaces of $M_n[\mathbf{R}]$; $\mathbf{P_n}[X] = \{a_0 + a_1X + \cdots + a_nX^n \ /\ a_i \in \mathbf{R}\ \forall\ 0 \le i \le n\}$ as a subspace of $\mathbf{R}[X]$ Eigen space associated to an eigen value as a subspace of $\mathbf{R}^n$ L(S) is a subspace of a vector space. The space of all solutions of the system of m homogeneous linear equations in n unknowns as a subspace of $\mathbf{R}^n$. Properties of a subspace such as necessary and sufficient condition for a non empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is a subset of the other	**[5L]**
2.3	Generating set, linearly independent/linearly dependent subsets of a vector space	**[2L]**
2.4	Basis and dimension of a vector space, applications	**[3L]**

Reference books

- S. Kumaresan; Linear algebra -A geometric approach; Prentice Hall of India(2000)
- Gilbert Strang; Linear Algebra; Wellesy-Cambridge Press

Additional Reference books:

- K Hoffman and R Kunze ; Linear algebra; Prentice-Hall
- Sheldon Axler; Linear Algebra Done Right; Springer

F. Y. B. Sc. (Mathematics)
SEMESTER II - Practicals
COURSE CODE: 23US2MTCCP Credits- 02

Learning Objectives:
The Practical is intended to
1. Solve problems based on the concepts learnt
2. Apply the concepts in various situations

Learning Outcome:
After the successful completion of the practical, the learner will be able to:

1. Apply the results proved to the other sciences
2. Create examples and counterexamples
3. Solve modern and classical problems

Module I	Calculus -2
1. Problems based on limits	
2. Problems based on continuity.	
3. Problems on differentiability	
4. Problems on mean value theorems and higher order derivatives	
5. Formal/Mathematical Proof writing such as: Chain rule, Inverse function theorem	

Module II	Linear Algebra -1
1. Problems based on matrices, geometric interpretation of system of equations	
2. To find the solution set of a given system of linear equations.	

3. Determine if the given set forms a vector space under given operations
4. Problems on subspaces
5. To find a basis and dimension of a vector space
6. Mathematical writing/ Formal Proof writing such as -a set is linearly dependent if and only if one of the elements is a linear combination of the other elements. Proving or disproving the converse
7. Mathematical writing/ Formal Proof writing such as -If the dimension of a subspace of a vector space is same as the dimension of the vector space , then the subspace is same as the vector space