K. J. SOMAIYA COLLEGE OF SCIENCE AND COMMERCE

AUTONOMOUS – Affiliated to University of Mumbai

Re-accredited “A’ Grade by NAAC
Vidyanagar, Vidyavihar, Mumbai 400 077

Syllabus for T.Y.B.Sc.

Programme: B.Sc.

Course : Mathematics

(Choice based Credit System

with effect from the academic year 2019–2020)

Syllabus -T.Y.B.Sc. Mathematics

Semester V										
Course No	Course Title	Course code	Credits	Hours	Periods (50 min)	Unit/ Module	Lectures (50 minutes) per module (app)	**Examination**		
								Internal Marks	External Marks	Total Marks
THEORY: Core courses										
I	Differential Calculus of several variable	20US5MTDC1	2.5	37.5	45	4	11	40	60	100
II	Modern Algebra	20US5MTLG2	2.5	37.5	45	4	11	40	60	100
III	Real Analysis and Metric Topology	20US5MTAT3	2.5	37.5	45	4	11	40	60	100
IV	Number Theory and its Application	20US5MTNT4	2.5	37.5	45	4	11	40	60	100
Practical on Core Courses										
I	Calculus and Algebra	20US5MTP1	2	60	72	2	20	___	_	100
II	Analysis and Number theory	20US5MTP2	2	60	72	2	20	_	_	100
Discipline Specific Electives (DSE)										
I	C programming	20US5MTCP5	2.5	37.5	45	4	11	40	60	100
II	C-Implementation of Mathematical Concepts	20US5MTCP6	2.5	37.5	45	4	11	40	60	100
I	Operation Research	20US5MTOR5	2.5	37.5	45	4	11	40	60	100

II	Practical Operational Research	20US5MTOR6	2.5	37.5	45	4	11	40	60	100
I	Graph Theory	20US5MTGT5	2.5	37.5	45	4	11	40	60	100
II	Application of Graph Theory	20US5MTGT6	2.5	37.5	45	4	11	40	60	100
Skill Enhancement Electives (SEC)										
I	SAGE, Maple and LaTeX	20US5MTSL7	2	60	72	2	20	_	_	100
TOTAL			20					280	420	1000

DSE II and Skill enhancement course are Practical oriented Courses

Semester VI										
Course No	Course Title	Course code	Credits	Hours	Periods (50 min)	Unit/ Module	Lectures (50 minutes)	**Examination**		
							per module (app)	Internal Marks	External Marks	Total Marks
THEORY: Core courses										
I	Integral Calculus	20US6MTIC1	2.5	37.5	45	4	11	40	60	100
II	Group theory and Ring Theory	20US6MTGR2	2.5	37.5	45	4	11	40	60	100

III	Metric Topology	20US6MTMT3	2.5	37.5	45	4	11	40	60	100
IV	Complex Analysis	20US6MTCA4	2.5	37.5	45	4	11	40	60	100
Practical on Core Courses										
I	Calculus and Algebra	20US6MTP1	2	60	72	2	20	_	_	100
II	Metric Topology and Complex Analysis	20US6MTP2	2	60	72	2	20	_	_	100
Discipline Specific Electives (DSE)										
I	JAVA programming	20US6MTJP5	2.5	37.5	45	4	11	40	60	100
II	JAVA-Implementation of Mathematical Concepts	20US6MTJP6	2.5	37.5	45	4	11	40	60	100
I	Operation Research	20US6MTOR5	2.5	37.5	45	4	11	40	60	100
II	Practical Operational Research	20US6MTOR6	2.5	37.5	45	4	11	40	60	100
I	Game Theory	20US6MTGT5	2.5	37.5	45	4	11	40	60	100
II	Application of Game Theory	20US6MTGT6	2.5	37.5	45	4	11	40	60	100
Skill Enhancement Electives (SEC)										
I	R Programming and LaTeX	20US6MTRL7	2	60	72	2	30	_	_	100
I	SQL and LaTeX	20US6MTDBL7	2	60	72	2	30	_	_	100
TOTAL			20					280	420	1000

Preamble

Mathematics is universally accepted as the queen of all sciences. This fact has been confirmed with the advances made in Science and Technology. Mathematics has become an imperative prerequisite for all the branches of science such as Physics, Statistics, and Computer Science etc. This proposed syllabus in Mathematics for Semester V and Semester VI of the B.Sc. Program aims to advance topics learnt in the previous four semesters.

At TYBSC we will deal with four core courses in mathematics, two discipline specific electives and one skill enhancement courses.

Course I will deal with Multivariable calculus with "Differential Calculus in several variable" in semester V and "Integral Calculus" in semester VI.

Course II will continue linear algebra learnt in from semester IV, Group theory and rings.

In course III they will learn sequence and series of real valued functions and topology of metric spaces.

In course IV students will learn number theory in semester V and Complex analysis in semester VI.

For Semester V and VI there are two elective discipline specific courses (DSE) and one skill enhancement course apart from the above four core courses. In semester V and VI a student will have an option of selecting courses specified in elective I or II or III. Elective I have two courses which are required for those set of students opting for IT field as their area of interest, Elective II have two courses useful for those interested in management field and Elective III will have two courses which are applied to both IT and Management fields apart from other areas of applications. Skill enhancement course is so designed that will help a student to understand topics learnt in Mathematics using mathematical software namely SAGE, Maple, Mathematics document editor LaTeX and R Programming/SQL.

Note:

Due to Workload restriction practical for DSE-I will be considered as DSE-II.

DSE-II being a practical oriented course and the evaluation of this course will be 40% internal having Projects and 60% semester end Practical examination of 2 hours duration.

In semester VI for skill enhancement, a student has an option to either select SQL or R Programming.

Skill enhancement courses will be evaluated in continuous evaluation basis and there will not be any end semester examination.

TYBSC (MATHEMATICS) Semester V

Theory Course I

Differential Calculus of Several Variables **Course Code:20US5MTDC1**

Title	Differential Calculus of Several Variables	
Course Code	20US5MTDC1	
Credit	2.5	
Module Number	Title of the Module	Number of lectures
Module 1	Vectors in $\mathbb{R}^n$	12
Module 2	Limits and Continuity of functions of several variables.	10
Module 3	Differentiability of scalar and vector valued functions.	11
Module 4	Mean value theorem, Taylor's theorem and applications.	12

Preamble

Calculus is the study of how things change. It provides a framework for modelling systems in which there is change, and a way to deduce the predictions of such models.

Multivariable calculus (also known as **multivariate calculus**) is the extension of calculus in one variable to calculus with functions of several variables: the differentiation and integration of functions involving several variables.
In mathematics, **differential calculus** is a subfield of calculus concerned with the study of the rates at which quantities change. It is one of the two traditional divisions of calculus, the other being integral calculus, the study of area, volume etc.

Differentiation has applications to nearly all quantitative disciplines. For example, in physics, the derivative of the displacement of a moving body with respect to time is the velocity of the body, and the derivative of velocity with respect to time is acceleration. The derivative of the momentum of a body with respect to time equals the force applied to the body; rearranging this derivative statement leads to the famous **F** = *m***a** equation associated with Newton's second law of motion. The reaction rate of a chemical reaction is a derivative.

In operations research, derivatives determine the most efficient ways to transport materials and design factories.

Derivatives are frequently used to find the maxima and minima of a function. Equations involving derivatives are called differential equations and are fundamental in describing natural phenomena. Derivatives and their generalizations appear in many fields of mathematics, such as complex analysis, functional analysis, differential geometry, measure theory, and abstract algebra.

Course Objective:

Students are expected to be able to Construct mathematical expressions and graphs involving functions and their derivatives.

Compute mathematical quantities using differential calculus and interpret their meaning.

Analyse mathematical statements and expressions.

Write logical progressions of precise statements to justify and communicate mathematical reasoning.

Apply calculus concepts to solve real-world problems such as optimization and related rates problems.

Course Outcome:

After learning the course, a student is expected to Graphically and analytically synthesize and apply multivariable and vector-valued functions and their derivatives, using correct notation and mathematical precision.

Module 1 Vectors in $\mathbb{R}^n$

Learning Objective:

The aim of this module is

To learn the structure of Euclidean n −dimensional space as a domain of functions of several variables.

To understand surfaces as graphs of functions.

Use of other coordinate systems such as polar coordinates, spherical and cylindrical coordinates.

Course Outcomes:

On completion of the course successful student will be able to:

Understand the vector structure of the Euclidean space, and effectively calculate the angles between lines, planes, measure lengths of vectors, area of parallelograms and volumes of parallelepiped.

Understand and be able to express surfaces as graphs or level surfaces of functions. Find the planes tangential to the surfaces and vectors normal to a surface at a given point.

Detailed Syllabus

Vectors in n-dimensional Euclidean space. Scalar product and norm of a vector. Angle between two vectors. Projection of a vector in direction of another vector. Cross product and scalar triple product in case of vectors in the three-dimensional space. Geometrical interpretation.

Lines and planes in $\mathbb{R}^3$. Angle between two lines and angle between two planes. Angle between a line and a plane. Distance between two parallel lines and distance between two parallel planes. Standard equations of quadric surfaces in the space, the sphere, ellipsoid, paraboloid, hyperboloids, cone, cylinder.

Polar coordinates in the plane. Cylindrical and spherical coordinates in the space.

Function of two variables taking real values. Its graph as a surface in the space. Level curves. Functions of three variables and level surfaces.

Module 2 Limits and Continuity of functions of several variables.

Learning Objective:

The aim of this module is to study concepts of limits and continuity in case of functions of several variables and understand its applicability.

Learning Outcomes:

On completion of the course, successful student will be able to:

apply the concepts of limits and continuity and be able to decide the existence of limits and evaluate these. Apply the same in a given situation.

Detailed syllabus

ε −neighborhood of a point in $\mathbb{R}^{\boldsymbol{n}}$. Scalar valued functions of two or more variables. Iterated limits and limit at a point in terms of $\varepsilon - \delta$. Use of polar coordinates and Two path test in the evaluation of limit. Continuity of scalar valued functions. Algebra of continuous functions.

Vector valued functions. Limits and continuity. A vector valued function $f(x) = \big(f_1(x), f_2(x), \ldots, f_m(x)\big)$ is continuous at x if and only if each of the scalar valued function $f_i(x)$ is continuous at x.

Module 3 Differentiability of scalar and vector valued functions.

Learning Objective:

The aim of this module is to:

Define and study the partial derivatives, directional derivatives, gradient of a scalar valued function of several variables.

To understand differentiation in terms of a linear transformation and its simplification in the form of gradient and Jacobian matrices.

To explore generalized analogues of chain rule, mean value theorem and Taylor's theorem.

Learning Outcomes:

On completion of the course successful student will be able to:

measure the rate of change along specific directions.

Detailed Syllabus

Partial derivatives of a scalar valued function at a point in $\mathbb{R}^{\boldsymbol{n}}$. Differentiation along a vector and directional derivative of a scalar valued function at a point. Gradient of a scalar valued function.

Definition of differentiability of a scalar valued function of several variables at a point in terms of a linear transformation. Uniqueness of the derivative as a linear transformation. Sum and product rule, Derivative of a scalar multiple of a differentiable function. Continuity of partial derivatives and relationship between Derivative of a scalar valued function and its gradient. Chain rule of differentiation in the form $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

For the composite function $z\big(x(t), y(t)\big)$, z a scalar valued function of two variables and $x(t), y(t)$ are functions of one variable.

Differentiation of vector valued functions. A vector valued function $f(x) = \big(f_1(x), f_2(x), \dots, f_m(x)\big)$ is differentiable at x if and only if each of the scalar valued function $f_i(x)$ is differentiable at x. Jacobian matrix of a differentiable function. Chain rule of differentiation and its verification using Jacobian matrices.

Module 4 Mean value theorem, Taylor's theorem and applications.

Learning Objective:

is module is to:

To explore the use of these concepts in finding approximate values, normal vectors to surfaces.

To use calculus to find the extreme values of functions.

Learning Outcomes:

On completion of the course successful student will be able to:

To use the model of functions of several variables in optimization techniques.

To Use the model of functions of several variables in finding approximate values of complicated expressions within specified error bounds

Detailed syllabus

Mean value theorem in case of scalar valued functions of several variables. Its illustration for two or three variables.

Linearization of a scalar valued function in the neighbourhood of a point. Computation of tangent planes and normal vectors to a surface using gradient.

Taylor's formula of first order and second order in case of scalar valued functions. Computation of approximate values using this formula.

Computation of Extreme values of a scalar valued function and saddle points using second derivative test.

Use of Lagrange multipliers to determine extreme values.

Recommended Books

1. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
2. M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995
3. G.B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
4. D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989
5. Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis (12th Edition, 1979), S. Chand and Co.

Activities / Projects

1. Sketching curves using polar coordinates.
2. Study of radial and transverse components of velocity and acceleration, tangential and normal components of acceleration in case of particle moving in a plane curve.
3. Study of conservative functions and obtaining the potential function using integration.

TYBSC (MATHEMATICS) SEMESTER V

Theory Course II

Linear Algebra and Group Theory **Course Code: 20US5MTLG2**

Title:	**Linear Algebra and Group Theory**	
Course Code:	**20US5MTLG2**	
Credits:	**2.5**	
Module Number	**Title of the Module**	**Number of lectures**
Module 1	Diagonalization	12
Module 2	Orthogonal diagonalization and Quadratic forms	11
Module 3	Groups and subgroups	12
Module 4	Cyclic Groups	11

Preamble

Linear Algebra is a continuous form of mathematics and is applied throughout science and engineering because it allows you to model natural phenomena and to compute them efficiently. Linear Algebra is central to almost all areas of mathematics like geometry and functional analysis. Its concepts are a crucial prerequisite for understanding the theory behind Machine Learning, especially if you are working with Deep Learning Algorithms. A good understanding of Linear Algebra will help one to make better decisions during a Machine Learning system's development. Linear algebra is vital in multiple areas of science in general. Practically every area of modern science contains models where equations are approximated by linear equations (using Taylor expansion arguments) and solving for the system helps the theory develop.

The concept of a group is central to abstract algebra: other well-known algebraic structures, such as rings, fields, and vector spaces, can all be seen as groups endowed with additional operations and axioms. Groups recur throughout mathematics, and the methods of group theory have influenced many parts of algebra. Linear algebraic groups and Lie groups are two branches of group theory that have experienced advances and have become subject areas in their own right.

Various physical systems, such as crystals and the hydrogen atom, may be modelled by symmetry groups. Group theory and the closely related representation theory have many important applications in physics, chemistry, and materials science. Group theory is also central to public key cryptography.

Course Objective: ,

The aim of this course is to introduce the concept of diagonalizable matrix, and orthogonal diagonalizable Matrix. Extend them to linear transformation. Study the Quadratic forms Understand groups and cyclic groups and associated properties.

Course outcomes:

At the end of the course, the learner will be able to check whether a matrix is diagonalizable and identify the given conic. The learner is able to verify and classify cyclic groups.

Module 1 Diagonalization of a matrix

Learning objectives:

The learner studies the concepts of diagonalization of a matrix and the criteria to test if a matrix is diagonalizable.

Learning outcomes:

At the end of the module, the learner will be able to find eigen values and eigen vectors of a matrix, and diagonalize the matrix if possible.

Detailed Syllabus

Definition of Eigen values and Eigen vectors of an n x n real matrix and a linear transformation T: V →V for a finite dimensional real vector space V, Characteristic polynomial of an n x n real matrix. Linear independence of eigen vectors corresponding to distinct eigen values, Eigen space. Similar matrices, similar matrices have same eigen values, Cayley Hamilton Theorem and its proof using relation between a matrix and its adjoint, applications of Cayley Hamilton Theorem.

Definition of diagonalizable square matrix. Matrix with distinct eigen values is diagonalizable, Algebraic multiplicity and Geometric multiplicity of an eigen value and their equality iff matrix is diagonalizable. Determining whether a square real matrix is diagonalizable, finding diagonalizing matrix.

Module II Orthogonal Diagonalization and Quadratic forms

Learning objectives:

The learner studies the concepts of diagonalization using an orthogonal matrix and use it to identify conic.

Learning outcomes:

At the end of module II, the learner will be able to orthogonally diagonalize symmetric matrix associated with a quadratic form and identify it.

Detailed Syllabus

Definition of Orthogonal diagonalization A real symmetric matrix has all real eigen values, and is orthogonally diagonalizable.

Application to real quadratic forms. Positive definite, semi-definite, indefinite matrices. Classification in terms of principal minors. Principle axes theorem. Classification of conics in $\mathbb{R}^2$ and quadric surfaces in $\mathbb{R}^3$.

Module III Groups and subgroups

Learning objectives:

The learner studies the definition and properties of groups and subgroups and when two groups are isomorphic.

Learning outcomes:

At the end of this module, the learner will be able to test if given set is a group or subgroup under the binary operation mentioned. Also the learner can identify isomorphic groups.

Detailed Syllabus

Review the definition and examples of a group. Abelian group. Order of a group, finite and infinite groups.

Permutation Group and Alternating Group.

Subgroups, definition and examples, N.S. condition for a non-empty subset to be a group. The Center Z(G) of a group G as a subgroup of G.

Cosets, Lagrange's theorem.

Group homomorphisms and isomorphisms. Examples and properties. Automorphisms of a group, characterization of automorphism of Z.

Module 4 Cyclic Groups

Learning objectives:

The learner studies the special type of groups viz cyclic group.

Learning outcomes:

At the end of this module, the learner will be able to test if given set is a cyclic group and the number of its generators.

Detailed Syllabus

Definition of a cyclic group, a finite group of order n is a cyclic group iff it contains an element of order n, subgroup of a cyclic group is cyclic. Examples of Z, Z_n, and μ_n as Cyclic groups.

Number of generators of a cyclic group, group of prime order is cyclic. Characterization of cyclic groups.

Recommended Books

1. S.Kumaresan. Linear Algebra: A Geometric Approach, Prentice Hall of India Pvt Ltd, New Delhi.
2. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
3. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
4. M. Artin, Algebra, Prentice Hall of India, New Delhi.
5. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.
6. L. Smith, Linear Algebra, Springer.
7. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
8. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
9. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books

1. S. Lang, Introduction to Linear Algebra, Second edition, Springer Verlag, New York.
2. K. Hoffman and S. Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
3. S. Adhikari. An Introduction to Commutative Algebra and Number theory. Narosa Publishing House.
4. T.W. Hungerford. Algebra. Springer.
5. D. Dummit, R. Foote. Abstract Algebra. John Wiley & Sons, Inc.
6. I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.

TYBSC (MATHEMATICS) SEMESTER V

Practical Course I

Calculus and Algebra **Course Code: 20US5MTP1**

Title:	Calculus and Algebra	
Course Code:	20US5MTP1	
Credits:	2.0	
Module Number	Title of the Module	Number of lectures
Module 1	Differential Calculus of several variables	20
Module 2	Linear Algebra and Group Theory	20

Module 1 Differential Calculus of several Variable

1. Problems on dot product, cross product, box product, calculating length of a vector, angle between lines, area of parallelograms, volume of parallelepiped. Finding equations of lines and planes in the space. Identifying standard quadric surfaces. Conversion between cartesian coordinates and polar, cylindrical and spherical coordinates. Sketching of simple curves in polar coordinates. Sketching level curves.
2. Problems on iterated limits, limits and continuity in case of scalar valued functions. Use of two path test and use of polar coordinates. Problems on limits and continuity in case of vector valued functions.
3. Computation of partial derivatives and directional derivatives. Use of gradient in computing the directional derivatives. Computing the derivative of a homogeneous function.
4. Computation of Jacobian matrices. Verification of chain rule. Verification of Mean value theorem in case of scalar valued functions.
5. Computing linearization of a scalar valued function of two or more variables. Tangential planes and normal vectors to surfaces at a given point.
6. Use of Taylor's second order formula in finding approximate values of complicated functions. Finding extrema and saddle points of a scalar valued function of two variables. Computing extreme values using Lagrange's multipliers.

Module 2 Linear Algebra and Group Theory

Finding eigen values and eigen vectors of a matrix, Applications of Cayley-Hamilton theorem.

Diagonalization of a matrix. Finding algebraic and geometric multiplicity.

Orthogonal diagonalization, identifying quadratic forms

Groups, Subgroups, Lagrange's Theorem

Group homomorphisms, isomorphisms

Cyclic groups

Additional practical using mathematical software like Maple or Maxima

1. Computations of limits, partial derivatives and gradients.
2. Plotting of 2D and 3D graphs. Observing the extreme values, local maxima, minima and saddle points.

TYBSC (MATHEMATICS) SEMESTER V

Theory Course III

Real Analysis and Metric Topology **Course Code: 20US5MTAT3**

Title:	Real Analysis and Metric Topology	
Course Code:	20US5MTAT3	
Credits:	2.5	
Module Number	Title of the Module	Number of lectures
Module 1	Sequence of functions	10
Module 2	Series of Functions	10
Module 3	Metric spaces	12
Module 4	Closed sets, Limit Points and Sequences	14

Preamble

Sequence and Series of function are topics in Mathematics that are essential to understand various areas of Mathematics such as Fourier series. One can study interesting problems in analysis, geometry etc if one understands the spaces of functions over X, and for this studying sequences and series of function helps. convergence of sequences of functions is at the core of mathematical statistics' two most foundational theorems. In quantum mechanics, the wavefunction of a particle can be expanded as an infinite sum of eigenfunctions of the Hamiltonian operator via the Spectral Theorem

Metric space methods have been employed for decades in various **applications**, such as, internet search engines, image classification, or protein classification, petroleum reservoir modelling. Metric spaces are generalization of Euclidean and Riemannian spaces that find direct application in mechanics and general relativity.

Course objective:

The aim of this course is to study convergence of Sequence and series of function. Understand the consequences of uniform. convergence of sequences of functions on differentiability and integrability.

Understand Open sets and closed sets in a metric space. Find the limit point of a set and its implications.

Understand Characterization of limit points and closure points in terms of sequences.

This leads to a better understanding of continuity, compactness, completeness and connectedness.

Course outcomes:

At the end of the course, the learner will be able to analyze the convergence of sequences and series of functions. The learner is able to identify metric spaces, open and closed sets and also find limit points of sets.

Module 1 Sequence of functions

Learning objectives:

The learner studies the concepts of convergence of sequences.

In the next Semester, the learner studies the advanced concepts such as continuity of functions from one metric space to another, compactness, completeness and connectedness.

Learning outcomes:

At the end of the module, the learner will be able to differentiate between pointwise and uniform convergence of sequences and also analyze the consequences of uniform convergence on continuity, integrability. series of functions.

Detailed Syllabus

Pointwise convergence of sequence of functions with examples

Uniform convergence of sequence of functions with examples

Uniform convergence implies pointwise convergence, example to show converse not true

Consequences of uniform. convergence of sequences of functions on Continuity with examples

Consequences of uniform. convergence of sequences of functions on Integrability with examples

Consequences of uniform. convergence of sequences of functions on differentiability with examples

M_n test for convergence of sequence of functions

Module 2 Series of Functions

Learning objectives: The learner comprehends the concepts of convergence of series of functions along with the consequences on continuity, integrabliity and differentiability.

Learning outcomes: At the end of the module, the learner will be able to differentiate between pointwise and uniform convergence of series and also analyzes term by term integration and differentiation.

Detailed Syllabus

convergence of series of functions

Weierstrass M-test. Examples

Consequences of uniform. convergence of series of functions on continuity

Consequences of uniform. convergence of series of functions on Integrability , examples

Consequences of uniform. convergence of series of functions on differentiability, examples

Term by term differentiation and integration.

Module 3 Metric spaces

Learning objectives: The student studies different types of metric spaces and concept of open sets., which help in the topics of the next semester.

Learning outcomes: At the end of the module the learner is able to identify metric spaces and open and closed sets and also find limit points of sets.

Detailed Syllabus

Definition, examples of metric spaces $\mathbb{R}$, $\mathbb{R}^2$, Euclidean space $\mathbb{R}^n$ with its Euclidean sup and sum metric,

$\mathbb{C}$ (complex numbers), the spaces ℓ^1 and ℓ^2 of sequences and the space, C[a,b], the space of real valued continuous functions on [a,b], Discrete metric space.

Distance metric induced by the norm; translation invariance of the metric induced by the norm.

Metric subspaces, Product of two metric spaces.

Open balls and open set in a metric space, examples of open sets in various metric spaces, Hausdorff property

Interior of a set, Properties of open sets, Structure of an open set in $\mathbb{R}$,

Equivalent metrics. Distance of a point from a set, between sets, diameter of a set in a metric space and bounded sets

Module 4 Closed sets, Limit Points and Sequences

Learning objectives: The learner comprehends closed sets, limit points and sequences in metric spaces.

Learning outcomes: The learner is able to identify closed sets and also find limit points of sets. This leads to a better understanding of continuity, compactness, completeness and connectedness

Detailed Syllabus

Closed ball in a metric space, Closed sets- definition, examples

Limit point of a set, Isolated point, A closed set contains all its limit points, Closure of a set and boundary

Sequences in a metric space, Convergent sequence in a metric space, Cauchy sequence in a metric space, subsequence, examples of convergent and Cauchy sequence in finite metric spaces, $\mathbb{R}^n$ with different metrics and other metric spaces.

Characterization of limit points and closure points in terms of sequences. Definition and examples of relative openness/closeness in subspaces, Dense subsets in a metric space and Separability

References Books:

1. Kumaresan, S. (2005) Topology of Metric spaces

2. Copson, E. T. (1996) Metric Spaces. Universal Book Stall, New Delhi. .

3. Goldberg, R. R.(1970) Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi

Additional Reference Books.

1. Rudin, W. (1976) Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland.

2. Apostol, T. (1974) Mathematical Analysis, Second edition, Narosa, New Delhi.

3. Jain, P. K., Ahmed, K. (1996) Metric Spaces. Narosa, New Delhi.

4. Somasundaram, D. , Choudhary,B. (1997)A first Course in Mathematical Analysis. Narosa, New Delhi

5. Simmons, G.F. (1963) Introduction to Topology and Modern Analysis, McGraw-Hii, New York .

TYBSC (MATHEMATICS) SEMESTER V

Theory Course IV

Number Theory and its Applications **Course Code: 20US5MTNT4**

Title:	Number Theory and its Applications	
Course Code:	20US5MTNT4	
Credits:	2.5	
Module Number	Title of the Module	Number of lectures
Module 1	Prime numbers and Congruences	11
Module 2	Pell's Equations, Arithmetic Functions and Special Numbers	12
Module 3	Quadratic Reciprocity.	11
Module 4	Cryptography	11

Preamble

Due to the unquestioned historical importance of number, the theory of numbers has always occupied a unique position in the world of mathematics. Number theory was labelled the "Queen of Mathematics " by Gauss. For many years it was thought that number theory does not have many practical applications. But this situation has changed drastically with rise of the computers. Prime and composite numbers play an significant role in modern cryptography and coding theory. An important application of number theory is keeping secrets the high volume of confidential information such as credit card number, bank account number etc.

Number theory is a broad subject with many strong connections with other branches of Mathematics. This subject teaches a student that how simply playing with numbers can lead you to the subject such as Group theory, Ring theory, Calculus etc. For this course not much of the prerequisite is required, even a student having just a basic knowledge of numbers and divisibility can learn this.

Course Objective: *The course aims to introduce some of the concepts in Number theory and solve many real life problems using number theoretical techniques .*

Learning outcome: *Student will be able to apply various theorems of Number Theory in solving problems. Student will be able to construct mathematical proofs of statements and find counterexamples to false statements in Number Theory.*

Module I Prime numbers and Congruence

Objective: *To prove some of the famous results involving congruence and to use them in solving problems.*

Learning Outcomes: *Student will be able to understand the logic and methods behind the major proofs in Number Theory. The student will be able to apply these theorems to solve problems related to congruence.*

Review: Divisibility, Definition of Prime, Definition and elementary properties of Congruence, The Fundamental Theorem of Arithmetic, Number of primes are infinite, Distribution of Primes (There are arbitrarily large gaps between consecutive primes).

Definition and examples of Twin primes, Complete Residue system modulo m, Reduced residue system modulo m, Euler's Phi Function, Euler's Phi function is multiplicative.

Fermat little Theorem, Euler's generalization of Fermat Little Theorem, Wilson Theorem.

Linear Congruence and its solution, Chinese Remainder Theorem, For prime p, $x^2 \equiv -1 (\text{mod } p)$ has a solution iff p is of the form 4k+1.

Module II Continued Fractions and Pell's Equations

Objective: *To introduce theory of continued fraction and its applications.*

Learning Outcomes: *Student will be able to solve Diophantine and Pell's equation using continued fraction. Student will be able to approximate an irrational with a rational with desired accuracy.*

Definition of finite continued fraction . Representation of a rational number as a finite simple continued fraction. Value of a finite continued fraction is always rational. Solving linear Diophantine equation using Continued Fraction.

K^{th} convergent (C_k) of a continued fraction. Representation of C_k as p_k / q_k. The convergents with even subscript forms a strictly increasing sequence and convergents with odd subscript forms a strictly decreasing sequence.

Definition of infinite continued fraction. Representation of an irrational number as an infinite simple continued fraction. Value of an infinite continued fraction is always irrational. Every irrational number has a unique representation as an infinite continued fraction. Rational approximation of an irrational number. If $1 \leq b \leq q_n$ then p_n/q_n is better rational approximation for irrational number x than any rational number a/b.

Definition of Pell's equation. Positive solution of a Pell's equation x^2-dy^2=1 is one of the convergent of continued fraction expansion of $\sqrt{d}$. Thue's Lemma. If n is the length of the continued fraction expansion of $\sqrt{d}$ then ${p_{kn-1}}^2 - d{q_{kn-1}}^2 = (-1)^{kn}$.

Module III Quadratic Reciprocity

***Objective**: To introduce the Legendre symbol and its applications.*

***Learning Outcomes**: Student will be able to compute Legendre and Jacobi symbol. Student will be able to decide about solvability of a quadratic congruence with prime modulus as well as composite modulus.*

Quadratic Residues , Euler's Criterion with proof. Legendre symbol and its properties.

Gauss Lemma, Quadratic Reciprocity Law.

There are infinitely primes of the form 4k+1, 8k-1, 6k+1. For a prime p characterising (2/p), (3/p), (5/p). Solving quadratic congruence equations.

The Jacobi Symbol and law of Reciprocity for Jacobi Symbol.

Module IV Cryptography

***Objective**: To introduce some classical methods to encrypt and decrypt a message.*

***Learning Outcomes**: Student will be able to encrypt and decrypt a message using classical cryptosystem.*

Classical Cryptosystems, Shift cipher, affine cipher, Vigenere cipher.

Play Fair cipher, ADFGX cipher, Block Cipher.

Public Key cryptosystem, RSA algorithm

Digital Signature: RSA signature

Recommended book:

David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw Hill Edition

Additional References:

1. H. Zuckerman and H. Montogomery. Elementary number theory. John Wiley & Sons. Inc.

2. G. H. Hardy, and E.M. Wright, An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.

3. wade Trappe, Lawrence C Washington Introduction to Cryptography with Coding Theory .

TYBSC (MATHEMATICS) SEMESTER V

Practical Course II

Analysis and Number Theory **Course Code: 20US5MTP2**

Title:	Analysis and Number Theory	
Course Code:	20US5MTP2	
Credits:	2.0	
Module Number	Title of the Module	Number of lectures
Module 1	Analysis	20
Module 2	Number Theory	20

Module 1 **Analysis**

1. Sequences of Functions
2. Series of Functions.
3. Examples of Metric Spaces, Open balls and Open sets in Metric / Normed Linear spaces, Interior Points
4. Subspaces, Closed Sets and Closure, Equivalent Metrics and Norms

5 Sequences, Convergent and Cauchy Sequences in a Metric Space

6 Limit Points, Diameter of a set, Dense Sets and Seperability

Module 2 Number Theory

1. Finding primes of given form, determining whether the given set is CRS and RRS modulo m, problems based on Euler's Phi function, Fermat's theorem and Euler's theorem.
2. Problems on based on Wilson theorem. Solving linear congruence and problems based on Chinese Remainder theorem.
3. Writing a rational number as simple finite continued fraction and finding a rational number represented by a finite simple continued fraction. Finding k^{th} convergent of a given finite continued fraction.
4. Finding the irrational number represented by an infinite simple continued fraction and writing an irrational number as infinite simple continued fraction. Finding

rational approximation to an irrational number with desired accuracy. Solving Pell's equations.

5. Computing Legendre symbols and solving quadratic congruence with prime modulus.
6. Computing Jacobi symbol and solving quadratic congruence with composite modulus.
7. Encryption and decryption using Shift cipher, affine cipher, Vigenere cipher, Play Fair cipher, ADFGX cipher and Block Cipher. Encryption and decryption using Public Key cryptosystem.

TYBSC (MATHEMATICS) SEMESTER V

Discipline Specific Elective Courses

Students are required to choose one of the three elective courses from Elective I/II/III.

Elective I

Preamble

C is a general-purpose programming language that is extremely popular, simple and flexible. It is machine-independent structured programming language. C is a base programming and is used to write operating systems (windows and many other) to complex programs like Oracle database, Git, Python interpreter and more. Using C, a student finds practical applications of many Math concepts, improve logical thinking capabilities and develop computer aided problem-solving capabilities.

Course Objective*: Introduce the basics of C programming and understand how it can be used to implement Mathematical concepts. and will be in a position to use the concepts learnt, in areas of data Science, Machine learning and other IT related fields in their future endeavor.*

Learning outcome*: Student will be in a position to implement C for various mathematical concepts*

Course code	Title
20US5MTCP5	C Programming
20US5MTCP6	C-implementation of Mathematical concepts

Elective II

Preamble

Operational research is an important tool used to solve complex problems. It is a quantitative approach that helps managers do their jobs effectively. Managers use techniques of OR to maintain better control over their subordinates as it helps to measure productivity, use it to identify problem areas and take corrective action. It involves various optimizing techniques.

Course Objective: *The course aims to introduce techniques for mathematical modelling and solve linear programming problems using graphical and Simplex methods.*

Learning outcome: *Student will be in a position to solve some of the linear programming problems and understand how it helps in management of productions, marketing etc.*

Course code	Title
20US5MTOR5	Operational Research
20US5MTOR6	Practical Operational Research

Elective III

Preamble

Graph theory began with recreational math problems and has grown into significant area of mathematical research with applications in Chemistry, Operation research, Social Sciences and Computer Science. It is a branch of Mathematics concerned with network points connected with lines and does not refer to data charts.

Course Objective: *The course aims to introduce some of the concepts in Graph theory and how, many of the real-life problems can be solved with little effort.*

Learning outcome: *Student will be in a position to solve some of the problems of networking, Scheduling and create some interesting games.*

Course code	Title
20US5MTGT5	Graph Theory
20US5MTGT6	Applications of Graph Theory

Discipline Specific Elective course- Elective I

TYBSC (MATHEMATICS) SEMESTER V

C Programming **Course code: 20US5MTCP5**

Title	C Programming	
Course code	20US5MTCP5	
Credits	2.5	
Module Number	Title of the Module	Number of lectures
Module I	Introduction to C Programming	15
Module II	Control statements	15
Module III	Functions in C	15

(Theory oriented course)

Module I Introduction to C Programming

Objective*: Introduce C concepts and arrays*

Learning Outcomes*: Student should be able to understand the syntax and use of various data type, Input/output statements, and arrays*

Structure of C program: Header and body, Concept of header files, Use of comments, Compilation of a program.

Data Concepts: Variables, Constants, data types like: int, float char, double and void. Qualifiers: short and long size qualifiers. Declaring variables, Scope of the variables according to block, Hierarchy of data types.

Types of operators: Arithmetic, Relational, Logical, Compound Assignment, Increment and decrement, Conditional or ternary operators. Precedence and order of evaluation. Statements and Expressions.

Type conversions: Automatic and Explicit type conversion.

Data Input and Output functions: Formatted I/O: printf(), scanf(). Character I/O format: getch(), gets(), putch(), puts().

Numeric functions such as pow(), sqrt(), etc.

Arrays: One-, two- and three-dimensional arrays.

declaring array variables, Array initialization, bound checking, accessing array elements**.**

Strings.

Module II Control statements

Objective*: Introduce conditional statements and various loops,*

Learning Outcomes*: Student should be able to understand the syntax and use of various branching statements and loops*

Iterations: Control statements for decision making: (a) Branching: if statement, if-else statement, if-else-if statement (b) Looping: while loop, do- while, for loop, nested loop. (c) Loop interruption statements: break, continue.

Switch statement.

Module III Functions in C

Objective*: Aim is to Introduce String functions and user defined functions.*

Learning Outcomes*: Student should be able to understand the syntax and use of various string functions and implement user defined simple and recursive functions.*

String Functions: strlen(), strcpy(), strcat(), strcmp().

User defined Functions: Function definition, return statement, calling a function.

Recursion: Definition, Recursion functions for factorial, Fibonacci sequence, exponential function, G.C.D.

Recommended Book:

(a) Let us C by Yashwant Kanetkar, BPB

Additional References:

(a) Programming in ANSI C (Third Edition) : E Balagurusamy, TMH
(b) Programming with C by Byron Gottfried, Schaum's outlines, Mc Graw Hill.
(c) Mastering Algorithms with C, Kyle Loudon, Shroff Publishers.
(d) Algorithms in C (Third Edition): Robert Sedgewick , Pearson Education Asia.
(e) Programming in ANSI C by Ram Kumar, Rakesh Agrawal, TMH.
(f) Programming with C (Second Edition): Byron S Gottfried (Adapted by Jitender Kumar Chhabra) Schaum's Outlines (TMH)
(g) Programming with C : K R Venugopal, Sudeep R Prasad TMH Outline Series.

SEMESTER V	
C-implementation of Mathematical concepts	**Course code: 20US5MTCP6**

Title	C-implementation of Mathematical concepts	
Course code	20US5MTCP6	
Credits	2.5	
Module Number	Title of the Module	Number of lectures
Module I	Illustrating the concepts of different iterations.	15
Module II	illustrating the concepts of one- and two-dimensional arrays.	15
Module III	illustrating the concepts of user defined functions, recursion.	15

(Practical oriented course. Evaluation will be 40% internal having Projects and 60% semester end Practical examination of 2 hours duration)

Objective: *The aim of this course is to introduce programming techniques and C implementation of various mathematical concepts.*

Learning outcome: *A student should be in a position write simple programs, implement mathematical concepts and debug simple programs.*

Write a C program that illustrates the concepts of different iterations.

1. Simple loops, nested loops, breaking out of a loop, skipping statements within a loop using continue statement.
2. Evaluating Sum of sequences
3 Extracting digits from a number and using it for different effect.
4 Finding LCM of two or more natural numbers.

Write a C program that illustrates the concepts of one- and two- dimensional arrays.

1. Array order reversal
2. Finding sum, sum of squares etc. Finding Standard deviation, Mean deviation from mean.
3. Sorting of numeric data. Finding median, mean deviation from median.
4. Finding maximum and minimum in a matrix, Row minimum and column minimum, Row sum and Column sum etc.

Write a C program that illustrates the concepts of functions, recursion.

1 Writing a user defined function using exponential, sine, cosine (etc.) series.

2. Using # define directive- calculating area, volume etc. of two- and three-dimensional objects.
3. Problems based on Financial mathematics.
4. Recursive function to calculate power of a number, Fibonacci sequence (and other sequences), factorial of a whole number, GCD of two or more natural numbers.

Discipline Specific Elective course- Elective II

TYBSC (MATHEMATICS) SEMESTER V	
Operational Research	**Course code: 20US5MTOR5**

Title	Operational Research	
Course code	20US5MTOR5	
Credits	2.5	
Module Number	Title of the Module	Number of lectures
Module I	Introduction to Operational Research	15
Module II	Linear Programming:	15
Module III	Simplex Method	15

(Theory oriented course)

Module I Introduction to Operational Research

Objective*: Introduce Operational research its scope and limitation and understand various stages of development of OR.*

Learning outcome*: A student will understand the origin and various development in OR. How Linear Algebra is applied to develop OR.*

Basics of Operational Research: Origin & Development of Operational Research, Definition and Meaning of Operational Research, Different Phases of an Operational Research Study, Scope and Limitations of Operational Research.

Six Stages of Development of Operations Research

I: Observe the problem environment

II: Analyze and define the problem

III: Develop a model

IV: Select appropriate data input

V: Provide a solution and test its reasonableness

VI: Implement the solution

Mathematical Modeling of Real-Life Problems.

Module II Linear Programming

Objective*: Aim of this module is to understand various concepts and use Graphical methods to solve Linear Programming problems*

Learning outcome*: A student after learning this module will be in a position to formulate and solve the linear programming problems graphically.*

Review of Linear algebra: Solution of a system of Linear Equations, Linear independence and dependence of vectors, Concept of Basis.

Basic Feasible solution, Convex sets. Extreme points, Hyperplanes and Halfspaces, Convex cones, Polyhedral sets and cones.

Linear Programming Problem Formulation, solution by Graphical Method, Graphical Solution of LPP- Bounded region, unbounded region, Use of Iso-Cost/profit lines to find solution

Module III Simplex Method

Objective*: Aim of this module is to understand various concepts and use Simplex method to solve Linear Programming problems*

Learning outcome*: A student after learning this module will be in a position to solve the linear programming problems using Simplex method, Two-Phase and Dual-Simplex method.*

Theory of Simplex Method, Simplex Algorithm, Two phase Method, Charnes-M Method, Degeneracy.

Theory of Duality, Dual-simplex method.

TYBSC (MATHEMATICS) SEMESTER V

Practical Operational Research **Course code: 20US5MTOR6**

Objective: *The aim of this course is to introduce linear programming techniques to solve various mathematical managerial problems.*

Learning outcome: *A student should be in a position solve simple linear programming problems using graphical, and simplex methods.*

Title	Practical aspect of Operational Research-I	
Course code	20US5MTOR6	
Credits	2.5	
Module Number	Title of the Module	Number of lectures
Module I	Formulating Linear Programming Problems and Solving Graphically	15
Module II	Formulating Linear Programming Problems and Solving it Using Simplex Methods and Charnes-M Method.	15
Module III	Problem Solving Using Two Phase and Dual-Simplex Method.	15

(Practical oriented course. Evaluation will be 40% internal having Projects and 60% semester end Practical examination of 2 hours duration)

Problem Solving

1. To solve Linear Programming Problem using Graphical Method with (i) Unbounded solution (ii) Infeasible solution (iii) Alternative or multiple solutions.
2. Solution of LPP with simplex method.
3. Problem solving using Charnes-M method.
4. Problem solving using Two Phase method and dual-simplex method.

5. Illustration of following special cases in LPP using Simplex method (i) Unrestricted variables (ii) Unbounded solution (iii) Infeasible solution (iv) Alternative or multiple solutions.

References /Suggested Readings:

1. G. Hadley: Linear Programming. Narosa, Reprint, 2002.

2. G. Hadley: Linear Algebra, Narosa, Reprint, 2002.

3. Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, 2010.

4. A. Ravindran, D. T. Phillips and James J. Solberg: Operations Research- Principles and Practice, John Wiley & Sons, 2005.

5. F.S. Hillier. G.J. Lieberman: Introduction to Operations Research- Concepts and Cases, 9th Edition, Tata Mc-Graw Hill, 2010.

Discipline Specific Elective course- Elective III

TYBSC (MATHEMATICS) SEMESTER V	
Graph Theory	**Course code 20US5MTGT5**

Title	Graph Theory	
Course code	20US5MTGT 5	
Credits	2.5	
Module Number	Title of the Module	Number of lectures
Module I	Graphs	15
Module II	Trees	15
Module III	Planar Graphs and Colouring in a Graph	15

(Theory oriented course)

Module I Graphs

***Objective**: The aim of this module is to understand some of the basic concepts in Graph theory and understand how it can be used to understand connectivity.*

***Learning outcome**: At the end of the module a student will be able to use Mengers theorem and apply it to certain graphs. Construct simple communication network.*

Review: Simple graphs, directed graphs, connectivity and matrix representation of graphs.

Laplacian Matrix, Degree. Special classes of graphs: Bipartite graphs, line graphs, chordal graphs.

Walks, trails, paths, connected graphs, Graphs isomorphism, operations on graphs, degree sequences.

Distance, cut vertices, cut-edges, blocks, connectivity, weighted graphs.

Vertex and Edge connectivity-Result $\kappa \leq \kappa 0 \leq \delta$, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.

Module II Trees

***Objective:** The aim of this module is to understand some of the concepts in trees and find minimal spanning tree with respect to weights and distance.*

***Learning outcome**: After the end of this module a student will be in a position to find the minimal spanning tree with respect to weights and distance between nodes. They will be able to traverse a tree and express a rooted tree as a mathematical expression and vice-a-versa.*

Characterizations of Trees, number of trees, minimal-spanning trees.

Cut vertices, cut edges, Bond, spanning trees, Fundamental cycles, Vector space associated with graph, Cayley's formula

Connector problem- Kruskal's algorithm, Proof of correctness, rooted trees and, Binary, m-ary trees.

Tree traversal (preorder, inorder, post order)

Huffman coding, Searching algorithms BFS and DFS algorithms.

Module III Planar graphs and Coloring in a graph

***Objective**: The aim of this module is to understand graph coloring, finding Chromatic polynomial and some of its application.*

***Learning outcome**: By the end of this module a student will be able to verify planarity of a graph. Find the minimum chromatic number. Find chromatic polynomial and understand the relation of roots and chromatic number vis-à-vis chromatic polynomial.*

Planar graph and Euler formula, 5 color theorem (without proof) four color theorem (without proof). In any simple connected planar graphs having f regions, n vertices and e edges the following inequalities hold:

$e \geq \frac{3}{2}f$ and $e \leq 3n - 6$. $K_{3,3}$ is not a planar graph.

Chromatic polynomial of some simple graph such as trees, cycles, complete graph wheel etc.

Chromatic equivalence of graphs. Isomorphic graphs are chromatically equivalent.

References:

1) Graph theory with application by J. A. Bondy and U. S. R. Murty

(Freely downloadable)

2) Graph Theory by Reinhard Diestel Electronic edition Springer Verlag. (Freely downloadable)

3) Graph theory with application, Narsingh DeoPrentice Hall publication

Application of Graph theory	Course code 20US5MTGT6

Objective: *The aim of this course is to introduce linear programming techniques to solve various mathematical managerial problems.*

Learning outcome: *A student should be in a position solve simple linear programming problems using graphical, and simplex methods.*

Title	Application of Graph theory	
Course code	20US5MTGT6	
Credits	2.5	
Module Number	**Title of the Module**	**Number of lectures**
Module I	Construction of network	15
Module II	Finding Minimal tree and tree traversal	15
Module III	Planarity and Chromatic polynomial	15

(Practical oriented course. Evaluation will be 40% internal having Projects and 60% semester end Practical examination of 2 hours duration)

1) A) Finding the adjacency and incident matrix of a given graph and drawing the graph for a given adjacency matrix / incident matrix.
   B) Walks and triangles in a graph.
   C) Graph isomorphism
2) A) Construction of network
   B) Application of Mengers theorem
3) A) Tree traversal
   B) Game tree
4) A) BFS and DFS tree
   B) Kruskal's algorithm
5) A) Planar graph and its application

B) Finding Chromatic number

C) Finding Chromatic Polynomial and relation to chromatic number

6) A) Chromatic number

B) Chromatic polynomial of simple graphs

Skill Enhancement Course

TYBSC (MATHEMATICS) SEMESTER V

Soft Skill – SAGE, Maple and LaTeX **Course code: 20US5MTSEML**

Title	Soft Skill -SAGE and Maple	
Course code	20US5MTSM7	
Credits	2	
Module Number	Title of the Module	Number of lectures
Module I	SAGE	15
Module II	Maple	15

(Practical oriented course. Evaluation will be continuous evaluation and there will not be any end semester examination)

Module I SAGE and Maple

Objective: *Understand the syntax and process to solve mathematical problems.*

Learning outcome: *The student will be in a position to use mathematical software to use in their research work.*

Solving Mathematical Problems based on

Linear Algebra, Algebra, Graph theory, Calculus etc.

Module II LaTeX

Objective: *Aim of this module is to use it for Documentation of mathematical work.*

Learning outcome: *After the module is learnt a student will be in a position to use it for certain aspect of the documentation.*

Documentation, Article writing, Table, Mathematical functions, Writing books, Beamer

TYBSC (MATHEMATICS) SEMESTER VI

Theory Course I

Course Title: Integral Calculus **Course Code: 20US6MTIC1**

Title	Differential Calculus of Several Variables	
Course Code	20US5MTIC1	
Credit	2.5	
Module Number	Title of the Module	Number of lectures
Module 1	Double and Triple integrals	11
Module 2	Change of variables and applications.	11
Module 3	Line Integral	12
Module 4	Surface Integral and Volume Integral	14

Preamble

Calculus is the study of how things change. It provides a framework for modelling systems in which there is change, and a way to deduce the predictions of such models.

Multivariable calculus (also known as **multivariate calculus**) is the extension of calculus in one variable to calculus with functions of several variables: the differentiation and integration of functions involving several variables.
In mathematics, **Integral calculus** is a subfield of calculus concerned with the study of finding area, volume etc.

Course Objective:

Compute mathematical quantities using integral calculus and interpret their meaning.

Analyse mathematical statements and expressions.

Write logical progressions of precise statements to justify and communicate mathematical reasoning.

Apply calculus concepts to solve real-world problems such as optimization and related rates problems.

Course Outcome:

After learning the course, a student is expected to know how to find area, volume etc.

Apply Green theorem, Stokes theorem **Gauss Divergence theorem** to replace the idea of integration along a contour by a suitable double integral or replace a surface integral by a simple volume integral

Syllabus:

Module 1 Double and Triple integrals

Learning Objective:

Review the concepts Riemann integration and extending these concepts to higher dimensions.

Study the technique of iterated integrals to evaluate the multiple integrals.

Course Outcomes:

On completion of the course successful student will be able to :

Understand and evaluate multiple integrals over typical regions in the plane as well as in the space.

Evaluate the length of a curve, area, volume of regions and quantities such as moment of inertia, work done, flow and flux etc which one frequently comes across in physics.

Detailed Syllabus

Review of Riemann integration.

Definition of double (resp: triple) integral of a function and bounded on a rectangle (resp:box). Geometric interpretation as area and volume. Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals.

Basic properties of double and triple integrals proved using the Fubini's theorem such as

(i) Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions. Formulae for the integrals of sums and scalar multiples of integrable functions.

(ii) Integrability of continuous functions.

(iii) Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains.

Module 2 Change of variables and applications.

Learning Objective

Learning how to use a change of variables so as to simplify the computations involved in the process of evaluating multiple integrals.

Learning Outcomes:

Understand and make use of a change of variables so as to simplify integration.

Detailed Syllabus

Change of variables formula (Statement only). Use of Polar coordinates in evaluation of double integrals. Simple examples using other changes of variables in double integration.

Cylindrical and spherical coordinates, and triple integration using these coordinates.

Differentiation under the integral sign.

Applications to finding the center of gravity and moments of inertia.

Module 3 Line Integral

Learning Objective

The aim of this module is to introduce parametrisation of curve, evaluate line integral. Apply Greens theorem

Learning Outcomes:

Appreciate the mathematical ideas of conservation principles and existence of potential function and relationship between these ideas.

Make use of these ideas in calculation of Line integral using antiderivatives.

Detailed Syllabus

Parametrization of a curve in the plane or in the space. Line integral of a scalar valued function. Finding length of a curve.

Line integral of a vector valued function. Application in Computation of work done and flow.

Green's theorem and its use in evaluation of line integrals, area of regions enclosed by a simple closed curve.

Conservative fields and path independence of the line integral, its equivalence with the existence of potential function. Applications.

Module 4 Surface Integral and Volume Integral

Learning Objective

Learning how to apply the techniques to evaluate quantities such as area, volume, mass, moment of inertia etc

Study the extension of the techniques of Riemann integration to integration along a curve in two or three dimensional space.

Study the extension of the techniques of Riemann integration to integration along a surface in three dimensional space.

To study the sophisticated Green's Stoke's theorems and Gauss' Divergence theorem and learn how to use these to replace the idea of integration along a contour by a suitable double integral or replace a surface integral by a simple volume integral.

Learning Outcomes:

Use sophisticated techniques of Green's theorem, stokes' theorem and Divergence theorem.

Detailed Syllabus

Parametrization of surfaces. Finding a normal vector. Expressing a given surface as a level surface and use of gradient to find the vector normal to the surface. Finding area of a surface,

Definition of surface integrals of scalar valued as well as of vector valued functions defined on a surface.

Curl and divergence of a vector field. Elementary identities involving gradient, curl and divergence.

Stokes Theorem (proof assuming the general from of Greens Theorem). Examples.
Gauss Divergence Theorem (proof only in the case of cubical domains). Examples.

References :

1. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
2. James Stewart , Calculus with early transcendental Functions
3. M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995
4. G.B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
5. D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989

Activities / Projects

Study of functions of bounded variation and rectilinear curves.
Replacing a function by polynomial of order 1 or 2 using Taylor's formula and obtaining approximate value of a double integral or a line integral and observing the error.

TYBSC (MATHEMATICS) SEMESTER VI

Theory Course II

Group Theory and Ring Theory **Course Code: 20US6MTGR2**

Title	Group Theory and Ring Theory	
Course Code:	20US6MTGR2	
Credits:	2.5	
Module Number	Title of the Module	Number of lectures
Module 1	Normal Subgroups	12
Module 2	Ring theory	12
Module 3	Integral domain and fields	11
Module 4	Factorization	11

Preamble

The concept of a group and Rings are central to abstract algebra. Groups recur throughout mathematics, and the methods of group theory have influenced many parts of algebra. Linear algebraic groups and Lie groups are two branches of group theory that have experienced advances and have become subject areas in their own right.

Various physical systems, such as crystals and the hydrogen atom, may be modelled by symmetry groups. Group theory and the closely related representation theory have many important applications in physics, chemistry, and materials science. Group theory is also central to public key cryptography.

Ring theory studies the structure of rings, their representations, or, in different language, modules, special classes of rings (group rings, division rings, universal enveloping algebras), as well as an array of properties that proved to be of interest both within the theory itself and for its applications, such as homological properties and polynomial identities.

Course Objective: ,

The aim of this course is to introduce the concept of Normal subgroups and how it can be used to create newer groups namely Quotient Group.

Learn ideal theory Equivalent to the idea of normal subgroup and study polynomial ring to some reasonable extent. Learn Unique factorization domain, Principal ideal domain and Euclidean domain.

Course outcomes:

At the end of the course, the learner will be able to classify groups up to order 7.

Find Maximal and Prime ideal. Construct integral domain and fields. Understand the relationship between Unique factorisation domain, Principal Ideal domain and Euclidean domain.

Module 1 Normal Subgroups

Learning objectives:

The learner will be knowledgeable of special type of sub group such as Normal Subgroups and understand the structure and characteristics of these subgroups.

Learning outcomes:

The learner will be able to distinguish normal Subgroups and use them to establish isomorphism of the groups using factor groups.

Detailed Syllabus

Normal subgroups of a group. Definition and examples including centre of a group.Quotient group, definition and examples.Alternating group A_n, cycles. Listing normal subgroups of A_4, S_3.

First, second and third Isomorphism theorem.

Cayley's theorem.

External direct product of a group. Properties of external direct products. Order of an element in a direct product, criterion for direct product to be cyclic.

Classification of groups of order ≤ 7.

Module 2 Ring theory

Learning objectives:

The learner will be conversant with rings in abstract algebra.

Learning outcomes:

At the end of the module, the learner will be able to write and verify criteria of ring use a combination of theoretical knowledge and independent thinking to investigate problem in ring theory and construct proofs.

Detailed Syllabus

Definition of a ring. (The definition should include the existence of a unity element.) Properties and examples of rings, including Z, Q, R, C, M_n(R), Q[X], R[X], C[X], Z[i] , Z[√2], Z[√−5], Z_n.

Commutative rings.

(i) Units in a ring. The multiplicative group of units of a ring.

(ii) Characteristic of a ring.

Ring homeomorphisms. First Isomorphism theorem of rings.Ideals in a ring, sum and product of ideals in a commutative ring.Quotient rings.

Module 3 **Integral domain and fields**

Learning objectives:

The learner studies special rings like Integral domain and fields.

Learning outcomes:
At the end of the module, the learner will be able to understand polynomial domains, quotient rings and Construction of quotient field of an integral domain.

Detailed Syllabus

Prime ideals and maximal ideals. Definition and examples. Characterization in terms of quotient rings.

Polynomial rings. Irreducible polynomials over an integral domain. Unique Factorization Theorem for polynomials over a field.

Integral domains and fields. Definition and examples.A finite integral domain is a field.

Characteristic of an integral domain, and of a finite field. Construction of quotient field of an integral domain (Emphasis on Z, Q).A field contains a subfield isomorphic to Zp or Q.

Module 4 Factorization.

Learning objectives:

The learner studies the concepts of special rings using factorization.

Learning outcomes:
At the end of the module, the learner will be able to differentiate between Euclidean domain (ED), Principal Ideal Domain (PID), Unique Factorization Domain (UFD.

Detailed Syllabus

Divisibility in an integral domain, irreducible and prime elements, ideals generated by prime and irreducible elements.

Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique Factorization Domain (UFD). Examples of ED: Z, $F[X]$, where F is a field, and Z[i].

An ED is a PID, a PID is a UFD. Prime (irreducible) elements in R[X], Q[X], Zp[X]. Prime and maximal ideals in polynomial rings.

Z[X] is not a PID. Z[X] is a UFD (Statement only).

Recommended Books
1. I.N. Herstein. Algebra.

2. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.M. Artin. Algebra.

3. N.S. Gopalkrishnan. University Algebra.

Additional reference

1. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
2. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.
5. L. Smith, Linear Algebra, Springer.
6. Tom M. Apostol, Calculus Volume 2, Second edition, John Wiley, New York, 1969.
7. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
8. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
9. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.
10. T.W. Hungerford. Algebra. Springer.
11. D. Dummit, R. Foote. Abstract Algebra. John Wiley & Sons, Inc.
12. I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.

TYBSC (MATHEMATICS) SEMESTER VI

Practical Course I

Calculus and Algebra **Course Code: 20US6MTP1**

Title:	Calculus and Algebra	
Course Code:	20US6MTP1	
Credits:	2.0	
Module Number	Title of the Module	Number of lectures
Module 1	Integral Calculus	20
Module 2	Group Theory and Ring Theory	20

Suggested Practical based on Course II, 20US5MT2

Module 1 Integral Calculus

1. Sketching region of integration for double and triple integral. Computation using iterated integrals. Change of order of integration.
2. Change of variables. Use of polar, cylindrical and spherical coordinates. Finding center of mass, moment of inertia.
3. Parametrization of a curve in two or three dimensions. Computation of line integral for scalar valued as well as vector valued functions. Finding length of a curve. Finding work done by a force, flow and flux.
4. Use of Green's theorem. Integration by change of parametrization. Computation of area of a region enclosed by a simple closed curve.
5. Conservative functions, potential function for a conservative function. Verification of path independence in case of a conservative function.
6. Parametrization of surfaces. Computation of vector normal to the given surface. Simple problems on computation of surface integrals. Using change of variable.
7. Divergence and curl. Problems on use of Stoke's theorem and Divergence theorem. Verification of Stoke's theorem and Divergence theorem.

Module 2 Group Theory and Ring Theory

1. Normal subgroups and quotient groups.

2. Cayley's Theorem and external direct product of groups.

3. Rings, Integral domains and fields.

4. Ideals, prime ideals and maximal ideals.

5. Ring homomorphism, isomorphism.

6. Euclidean Domain, Principal Ideal Domain and Unique Factorization Domain

Additional practicals using mathematical softwares like Mapple or Maximaor SAGE
Observing the shapes of various curves and surfaces.
Obtaining values of line integral, double and triple integration. Verification in case of simple problems.

TYBSC (MATHEMATICS) SEMESTER VI

Theory Course III

Metric Topology **Course Code : 20US6MTMT3**

Title:	Metric Topology	
Course Code:	20US6MTMT3	
Credits	2.5	
Module Number	Title of the Module	Number of lectures
Module 1	Continuity	12
Module 2	Compactness	12
Module 3	Complete metric spaces	12
Module 4	Connected metric spaces	12

Module 1 Continuity

Learning objectives: The student studies continuous functions and uniformly continuous functions from one metric space to another along with the characterization of continuous functions in terms of open and closed sets.

Learning outcomes: At the end of the module the learner is able to understand continuity in terms of open and closed sets and also apply this knowledge to understanding of Urysohn's Lemma.

Detailed Syllabus

$\varepsilon - \delta$ definition of continuity at a point of a function from one metric space to another.

Characterization of continuity at a point in terms of sequences, open sets. Characterization in terms of inverse image of open sets and closed sets.

Algebra of continuous real valued functions.Uniform continuity in a metric space, definition and examples (emphasis on $\mathbb{R}$).

Urysohn's Lemma

Module 2 Compactness

Learning objectives: The student analyzes compact sets, their characterization and applies this knowledge to understand the Heine Borel Theorem.

Learning outcomes: At the end of the module the learner is able to identify compact sets and appreciate the application to Important results like Bolzano-Bolzano-Weierstrass property, Sequentiallycompactness property.

Detailed Syllabus

Definition of a compact set in a metric space using an open cover Examples of compact sets in different metric spaces $\mathbb{R}$. ,$\mathbb{R}^2$,$\mathbb{R}^3$

Properties of compact sets such as compact set is closed and bounded, every infinite bounded subset of a compact metric space has a limit point, Heine Borel theorem- every subset of Euclidean metric space $\mathbb{R}^n$ is compact if and only if it is closed and bounded.a continuous function on a compact set is uniformly continuous, continuous image of a compact set is compact

Characterization of compact sets in: The equivalent statements for a subset of $\mathbb{R}^n$ to be compact

Bolzano-Weierstrass property, Sequentially compactness property.

Module 3 Complete metric spaces

Learning objectives: The student studies complete metric spaces which leads to the understanding of Cantors Intersection Theorem, Density theorem, Bolzano Weierstrass's Theorem and Intermediate Value Property.

Learning outcomes: At the end of the module the student will be able to understand Complete Metric Spaces and its applications.

Detailed Syllabus

Definition of complete metric spaces, Examples of complete metric spaces. Completeness property in subspaces.

Nested Interval theorem in $\mathbb{R}$. Cantor's Intersection Theorem. Applications of Cantors Intersection Theorem: The set of real numbers is uncountable, Density of rational numbers (Between any two real numbers there exists a rational number), Bolzano Weierstrass Theorem: Every bounded sequence of real numbers has a convergent subsequence, Intermediate Value theorem: Let f:[a,b]→ $\mathbb{R}$. be continuous, and assume that f(a) and f(b) are of different signs say, f(a) < 0 and f(b) > 0. Then there exists c ϵ [a,b] such that f(c) =0.

Heine Borel theorem: Let I= [a,b] be a closed and bounded interval and let $\{J_\alpha / \alpha \in \Lambda\}$ be a family of open intervals such that $J \subset \cup_{\alpha\in\Lambda} J_\alpha$ Then there exists a finite subset $F \subset \Lambda$ such that $J \subset \cup_{\alpha \in F} J_\alpha$, that is, F is contained in the union of a finite number of open intervals of the given family. Finite intersection property of closed sets for compact metric space, hence every compact metric space is complete

Module 4 Connected metric spaces

Learning objectives: The student analyzes connected sets and their characterization and also path connectedtness.

Learning outcomes: At the end of the module the student will be able to understand connectedness and path connectedtness.

Detailed Syllabus

Connected sets Definition and examples.

Characterization of a connected space, namely a metric space X is connected if and only if every continuous function from X to {-1,1} is a constant function.

A continuous image of a connected set is connected.

(i) Path connectedness in $\mathbb{R}^n$, definitions and examples.

A path connected subset of $\mathbb{R}^n$ is connected.

An example of a connected subset of $\mathbb{R}^n$ which is not path connected.

Recommended Books:

Kumaresan, S. (2005) Topology of Metric spaces

Copson, E. T. (1996) Metric Spaces. Universal Book Stall, New Delhi. .

Additional Reference Books.

Rudin, W. (1976) Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland.

Apostol, T. (1974) Mathematical Analysis, Second edition, Narosa, New Delhi.

Jain, P. K., Ahmed, K. (1996) Metric Spaces. Narosa, New Delhi.

Somasundaram, D. , Choudhary,B. (1997)A first Course in Mathematical Analysis. Narosa, New Delhi

Simmons, G.F. (1963) Introduction to Topology and Modern Analysis, McGraw-Hill, New York .

TYBSC (MATHEMATICS) SEMESTER VI

Theory Course IV

Complex Analysis **Course Code: 20US6MTCA4**

Course code	20US6MTCA4	
Title	Complex Analysis	
Credits	2.5	Teaching load
Module I	Analytic Functions	12
Module II	Integrals	11
Module III	Cauchy integral formula and power series	11
Module IV	Singularities	11

Preamble

Complex numbers and complex analysis show up everywhere in mathematics and physics. Topologically Complex plane looks very much like $\mathbb{R}^2$. Algebraically, complex numbers are closed. This is a good algebraic property for a field. They've been studied in mathematics since the 17th century because of their applications to mathematics, mechanics, waves, etc. Complex numbers show up in number theory a lot. From the analytical point of view, there is a beautiful theory for series in complex analysis. Residues are a powerful tool for computation of integrals. Euler's identity shows us that logarithms, exponential functions and trigonometric/hyperbolic functions and their inverses can be thought of in a unified way. Winding numbers are important objects of study in Algebraic topology, but actually the first ideas of using them came from complex analysis. And many many other reasons that people can say why complex analysis is an important, and also beautiful, branch of mathematics.

Course Objective: *To study how a complex function is differentiated and integrated and how it differs from that of real valued functions. To study power series of complex numbers and different types of singularities.*

Learning outcome: *Student will be able to compute derivative and integration of complex valued functions. Student will be able to identify the singularity. Student will be able to apply*

Module I Analytic Functions

***Objective**: To find necessary and sufficient condition for differentiability of a complex function and to prove algebra of differentiation.*

***Learning Outcomes**: Student will be able to decide about differentiability of a complex function using Cauchy-Riemann equations. Student will be able to find the derivative of a complex function if it exists.*

Detailed Syllabus

Functions of Complex Variables, Limits, Theorems on limits, Limits involving the point at infinity.

Continuity, Derivatives, Differentiation formulas.

Cauchy-Riemann Equations, Sufficient Conditions for differentiability, Polar coordinates.

Analytic functions, f, g analytic then f + g, f − g, fg and f /g are analytic, chain rule, Harmonic functions.

Theorem: If f(z) = 0 everywhere in a domain D, then f(z) must be constant throughout D Harmonic functions and harmonic conjugate.

Module II Integrals

***Objective**: To study logarithm of complex number and observe how it is different from logarithm of real numbers. To find contour integral of a complex function.*

***Learning Outcomes**: Student will be able to find principal and general branch of logarithm of a complex number. Student will be able to find the contour integral of a function.*

Detailed Syllabus

Review of the Exponential functions, the Logarithmic function, Complex exponents, Trigonometric functions, Hyperbolic functions.

Branches and derivatives of logarithms, Some identities involving logarithms.

Derivatives of functions, Definite integrals of functions, Contours, Contour integral, Examples.

Upper bounds for Moduli of contour integrals, Anti-derivatives, Examples, Cauchy-Groursat's Theorem (without proof), Simply and multiply Collected domains.

Module III Cauchy integral formula and Power series

***Objective**: To introduce power series of complex numbers.*

***Learning Outcomes**: Student will be able to apply Cauchy integral formula to find line integral. Student will be able to find radius of convergence of a power series.*

Detailed Syllabus

Explain how to evaluate the line integral $\int f(z)\,dz$ over $|z-z_0| = r$ and prove the Cauchy integral formula : If f is analytic in $B(z_0, r)$ then for any w in $B(z_0, r)$ we have $f(w) = \frac{1}{2\pi i}\int \frac{f(z)}{z-w}dz$, over $|z - z_0| = r$.

Taylors theorem for analytic function , Mobius transformations: definition and examples.

Power series of complex numbers definition and examples.

Radius of convergences, disc of convergence, uniqueness of series representation, examples.

Module IV Singularities

Objective*: To find Taylor's and Laurent series of a complex function.*

Learning Outcomes*: Student will be able to find Taylor's and Laurent series of a complex function. Student will be able to point out the singularities. Student will be able to categorise the singularity as removable or isolated singularity. Student will be able to find residues and poles of a complex function.*

Detailed Syllabus

Taylor's series, Laurent series (without proof), example

Isolated singular points, Residues, Cauchy residue theorem, residue at infinity.

Types of isolated singular points, residues at poles, zeros of analytic functions, zeros and poles.

Main Reference:

(a) J.W. Brown and R.V. Churchill, Complex Variables and Applications, International Student Edition, 2009. (Eighth Edition).

Additional References:

1. S. Ponnusamy, Complex Analysis, Second Edition (Narosa).
2. S. Lang, Complex Analysis, (Springer Verlag).
3. A.R. Shastri, An Introduction to Complex Analysis, (MacMillan).
4. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable.

TYBSC (MATHEMATICS) SEMESTER VI

Practical Course II

Calculus and Algebra **Course Code: 20US6MTP2**

Title:	Metric Topology and Complex Analysis	
Course Code:	20US6MTP2	
Credits:	2.0	
Module Number	Title of the Module	Number of lectures
Module 1	Metric Topology	20
Module 2	Complex Analysis	20

Module 1 Metric Topology

1. Continuous functions on metric spaces
2. Uniform continuity, fixed point theorem
3. Examples of Compact Sets
4. Examples of complete metric spaces
5. Cantor's Theorem and Applications
6. Examples of connected sets and connected metric spaces . Path connectedness, convex sets, equivalent condition for connected set using continuous function

Module 2 Complex Analysis

1. Analytic Functions
2. Integrals
3. Cauchy integral formula
4. power series in R
5. Power Series in C
6. Singularities

TYBSC (MATHEMATICS) SEMESTER VI

Discipline Specific Elective Courses

Students are required to select courses from one of the electives I/II/III

Elective I

Preamble

Java is a powerful, high level, object-oriented programming language. There are a lot of advantages of learning Java. It is a perfect moment for a student to get to understand the concepts like OOP which is, one of the advanced programming concepts after familiarizing themselves with C (or C++).In addition to that, Java is being used for Web, Mobile(Android), embedded systems, big data and many other areas in the IT related field. Some real-life examples where java is being used are Minecraft, all android apps, Web applications which are using Spring & Struts, Trading application Murex, Hadoop, IDEs like Eclipse, Netbeans & Intellij etc.

Course Objective:

1. Understand fundamentals of programming such as variables, conditional and iterative execution, methods, etc.

2. Understand fundamentals of object-oriented programming in Java, including defining classes, invoking methods, using class libraries, etc.

3. Be aware of the important topics and principles of software development.

4. Have the ability to write a computer program to solve specified problems.

5. Be able to use the Java SDK environment to create, debug and run simple Java programs.

***Learning outcome**: After learning basics of programming with a modern programming language, Java. The student will be able to Design, develop and test Java programs using classes, methods, conditionals, loops, etc. Student will be in a position to implement Java for various mathematical concepts and will be in a position to use the concepts learnt, in areas of data Science, Machine learning and other IT related fields in their future endeavor.*

Course code	Title
20US6MTJP5	Java Programming
20US6MTJP6	Java-implementation of Mathematical concepts

Elective II

Preamble

Operational research is an important tool used to solve complex problems. It is a quantitative approach that helps managers do their jobs effectively. Managers use techniques of OR to maintain better control. The **Transportation** and **Assignment problems** deal with **assigning** sources and jobs to destinations and machines. It involves various optimizing techniques.

Course Objective: *The course aims to introduce techniques to mathematical modelling and solve linear programming problems of transportation and assignment.*

Learning outcome: *Student will be in a position to solve transportation and assignment problems and understand how it helps in management minimize cost of transportation and assigning jobs to minimize time/cost or maximize profit.*

Course code	Title
20US6MTOR5	Operational Research-II
20US6MTOR6	Practical aspect of Operational Research-II

Elective III-DSE I

Preamble

Game theory is a branch of Mathematical Economics that studies strategic interactions amongst rational decision makers. Traditionally, game theoretic tools have been applied to solve problems in Economics, Business, Political Science, Biology, Sociology, Computer Science, Logic, and Ethics. In recent years, applications of game theory have been successfully extended to several areas of engineered / networked system such as wireline and wireless communications, static and dynamic spectrum auction, social and economic networks.

Course Objective: *This course is intended to provide students with a comprehensive treatment of game theory with specific emphasis on applications in Economics and Engineering.*

Aim The aim of this course is to introduce students to the novel concepts of Game Theory with special emphasis on its applications in diverse fields and current research.

Learning outcome: *Game theory is the foundation of modern microeconomics. The students will have an understanding of the strategic interactions of game theory that have to be applied to address questions such as:*

• How, markets are more competitive than monopoly, but less competitive than perfect competition work? • How will entrants to the labor force differentiate themselves when quality is hard for firms to observe? • Why does Costco pay an average wage of $21/hour, when it is obvious that this is far more than is necessary to attract competent workers?

• What type of auction generates the most revenue for sellers? • How can we explain the proliferation of McDonald's and similar restaurants when everyone knows there are better restaurants?

Course code	Title
20US6MTGMT5	Game Theory
20US6MTGMT6	Practical approach to Game Theory

TYBSC (MATHEMATICS) SEMESTER VI

Discipline Specific Elective course- Elective I

JAVA Programming **Course code: 20US6MTD1JP**

Title	JAVA Programming	
Course code	20US6MTJP5	
Credits	2.5	
Module Number	Title of the Module	Number of lectures
Module I	Introduction to JAVA	15
Module II	JAVA: INHERITANCE	15
Module III	JAVA: APPLETS	15

Module I **Introduction to JAVA**

Objective: *Understand the difference in structured programming and object-oriented programming, Features of OOPs. Accepting data from the command prompt. Creating classes and objects. Understanding various data types and their conversion to another data type. Understand Some special methods its uses and understand arrays. Understand difference in Java's approach to arrays as compared to C.*

Learning Outcome*: At the end of learning the module, a student is expected to write simple programs, implement Java for various mathematical concepts learnt.*

Object-Oriented approach: Comparison between structured and object-oriented approach. Features of object-orientations: Abstraction, Inheritance, Encapsulation and Polymorphism. Concept of package. Integer class method: parseInt().

Introduction: History of Java, Java features, different types of Java programs, Differentiate Java with C. Java Virtual Machine.

Java Basics: Variables and data types, declaring variables, literals: numeric, Boolean, character and String literals, keywords, type conversion and casting. Standard default values. Java Operators, Loops and Controls.

Classes: Defining a class, creating instance and class members: creating object of a class; accessing instance variables of a class; creating method; naming method of a class; accessing method of a class; 'this' keyword, constructor Basic Constructor; parameterized constructor; calling another constructor. Finalizer method (only concepts)

Arrays: one and two-dimensional array, declaring array variables, creating array objects, accessing array elements.

Module II JAVA: INHERITANCE

Objective: *Introduce Inheritance its scope and limitations. Implement them for various mathematical concepts.*

Learning outcome: *After learning this module, a student should be able to write programs involving inheritance and include overloading and overriding methods for various instances.*

Inheritance: Various types of inheritance, super and subclasses, keywords- 'extends'; 'super', final.

overloading methods

overriding method

Module III JAVA: APPLETS

Objective: *The aim of this module is to introduce concepts of Applete which can be used for various effects.*

Learning outcome: *A student after learning this module should be able to draw complex animated figures using simple geometric structures.*

JAVA Applets: Difference of applet and application, creating applets, applet life cycle.

Graphics, Fonts and Color: The graphics class, painting, repainting and updating an applet, sizing graphics. Font class, draw graphical figures - lines and rectangle, circle and ellipse, drawing arcs, drawing polygons. Working with Colors: Color methods, setting the paint mode.

Main Reference:

(a) Java The Complete Reference, 8th Edition, Herbert Schildt, Tata McGraw Hill

Additional References:

(a) Programming with Java: A Primer 4th Edition by E. Balagurusamy, Tata McGraw Hill.
(b) Eric Jendrock, Jennifer Ball, D Carson and others, The Java EE 5 Tutorial, Pearson Education, Third Edition, 2003.
(c) Ivan Bayross, Web Enabled Commercial Applications Development Using Java 2, BPB Publications, Revised Edition, 2006
(d) Joe Wigglesworth and Paula McMillan, Java Programming: Advanced Topics, Thomson Course Technology (SPD), Third Edition, 2004.
(e) The Java Tutorials of Sun Microsystems Inc. http://docs.oracle.com/javase/tutorial

TYBSC (MATHEMATICS) SEMESTER VI

JAVA implementation of Mathematical concepts **Course code: 20US6MTD2JP**

Title	JAVA implementation of Mathematical concepts	
Course code	20US6MTJP6	
Credits	2.5	
Module number	Title	Number of lectures
Module I	Illustrating one-, two-dimension arrays	15
Module II	Illustrating Two-class programme	15
Module III	Illustrating the concepts of inheritance	15

(Practical oriented course. Evaluation will be 40% internal having Projects and 60% semester end Practical examination of 2 hours duration).

Write a Java program that illustrates the concepts of one, two-dimension arrays.

1. Generating sequence defined recursively.
2. Matrix addition, transpose
3. Matrix multiplication
4. Finding number of triangles in a graph

Two-class programming

1. Write a Java program that illustrates the concepts of Java class that includes (a)constructor with and without parameters.
2. Finding factors, prime factorization,
3. Verifying prime number
4. Divisibility tests of 2, 3, 4, 5, 9, 11. Finding leap year

Write a Java program that illustrates the concepts of inheritance including programs involving Overloading and overriding methods.

1. Programs involving subclass constructor calling constructor of the super class.
2. Overloading problems:

GCD of two and more numbers, common factors of two or more numbers, Perfect numbers, amicable numbers, Mersenne primes etc.

3. Overriding problems:
perimeter to area, Surface area to volume, simple interest to compound interest etc.

4. Writing JAVA Applet to draw geometric figures.

Discipline Specific Elective course- Elective II

T.Y.B.Sc. (Mathematics) SEMESTER VI	
Operational Research	**Course code: 20US6MTOR5**

Title	Operational Research	
Course code	20US6MTOR5	
Credits	2.5	
Module Number	Title	Number of lectures
Module I	Transportation problem	15
Module II	Solving Transportation and Transshipment problem	15
Module III	Assignment problem	15

(Theory Oriented Course)

Module I Transportation problem

Objective: Introduce the concept of Transportation problem. Initial allocation using few methods.

Learning Outcome: A student should be in a position to formulate and find basic feasible solution using North-West corner rule, Matrix minimum rule and Voggle's Approximation method to find the initial basic feasible solution.

Introduction and formulation

Balance and unbalanced transportation problem

Basic feasible solution

North-West Corner rule, Row minimum, Matrix minimum, VAM to find IBFS

Module II Solving Transportation and Transshipment problem

Objective: *The aim of this module is to introduce Modified distribution method to optimize cost/profit in a transportation kind of problem. Understand Transshipment and method to solve it.*

Learning outcome*: After learning this module a student should be able to solve a transportation problem with constraints.*

Modified Distribution method

Maximization of problem

Problems with minimum allotment and maximum allotment constraints.

Formulating and solving Transshipment problems

Module III Assignment problem

Objective*: Aim of this module is to understand Assignment problem and ways to solve it.*

Learning Outcome*: After learning this module a student is expected to formulate and solve Assignment problem with or without constraints other than cost/ profit.*

Introduction

Assignment Problem Structure and Solution

Unbalanced Assignment Problem

Infeasible Assignment Problem

Maximization in an Assignment Problem

Crew Assignment Problem

References /Suggested Readings:

1. G. Hadley: Linear Programming. Narosa, Reprint, 2002.

2. G. Hadley: Linear Algebra, Narosa, Reprint, 2002.

3. Hamdy A. Taha: Operations Research-An Introduction, Prentice Hall, 9th Edition, 2010.

4. A. Ravindran, D. T. Phillips and James J. Solberg: Operations Research- Principles and Practice, John Wiley & Sons, 2005.

5. F.S. Hillier. G.J. Lieberman: Introduction to Operations Research- Concepts and Cases, 9th Edition, Tata Mc-Graw Hill, 2010.

T.Y.B.Sc. (Mathematics) SEMESTER VI	
Operational Research	**Course code: 20US6MTOR6**

Title	Operational Research	
Course code	20US6MTOR6	
Credits	2.5	
Module Number	Title	Number of Lectures
Module I	Finding IBFS and Solving Transportation problems	15
Module II	Solving Transportation with additional constraints and Transshipment problem	15
Module III	Solving Assignment problem	15

(Practical oriented course. Evaluation will be 40% internal having Projects and 60% semester end Practical examination of 2 hours duration).

Objective: *Aim of this course is to formulate and solve transportation and Assignment and related problems using various optimizing techniques.*

Learning Outcome*: After learning this course a student will be in a position to solve transportation and Assignment and related problems.*

Module I	Finding IBFS and Solving Transportation problems
Module II	Solving Transportation with additional constraints and Transshipment problem
Module III	Solving Assignment problem

Discipline Specific Elective course- Elective III

T.Y.B.Sc. (Mathematics) SEMESTER VI	
Game Theory	**Course code: 20US6MTGMT5**

(Theory Oriented Course)

Title	Game Theory	
Course code	20US6MTGMT5	
Credits	2.5	
Module Number	Title	Number of Lectures
Module I	Analysing games:	15
Module II	Principle of Dominance	15
Module III	Methods to solve Zero-sum games	15

Module I Analysing games:

***Objective**: Aim of this module is to introduce Game theory, its Characteristics, And zero-sum game*

***Learning Outcome**: A student after learning this module is expected to solve simple competitive Games graphically.*

Competitive games, Characteristic of competitive games,

zero-sum and non-zero-sum games, Two-persons Zero-sum games, conservative players, saddle point and value of a game.

Maximin-Minimax criterion, Games without saddle points,

Graphic method for 2xn and mx2 Games.

Combinatorial games
General-sum games, Nash equilibria

Module II Principle of Dominance

***Objective**: Understand the Principles of solving Zero-sum games.*

***Learning outcomes**: A student after learning this course is expected to understand the principle behind zero-sum games and how one can solve them.*

Symmetric game, Minimax and saddle point theorem,

Fundamental theorem of matrix Game,

Principle of Dominance

Module III Methods to solve Zero-sum games

***Objective**: Aim of this module is to introduce some of the methods that can be employed to solve a zero-sum problem.*

***Learning outcome**: After learning this module students are expected to know how to solve some of the zero-sum problems.*

Connection between Game and LP. Algebraic method for m x n Games, Iterative method for approximate solution, Extension of two person games.

References Sultan chand and sons

1. Linear Programming and theory of games, P.K. Gupta and Manmohan

2. Game Theory, Thomas S. Ferguson.

3. Essentials of Game Theory Kevin Leyton-Brown and Yoav Shoham.

4. gametheory.net

5. (IGT) Martin Osborne, An Introduction to Game Theory, Oxford University Press, 2003

6. (AT) Vijay Krishna, Auction Theory, Academic Press.

7. (SG) PrajitDutta, Strategies and Games, MIT Press

8. (Website 1) http://www.ece.stevens-tech.edu/~ccomanic/ee800c.html

9. (GTWE) Allan MacKenzie, Game Theory for Wireless Engineers, Synthesis lectures on Communications, 2006

10. (IITD Website)

11. (HV) Hal Varian, Microeconomic Analysis, Norton

12. (Gandhi) Gandhi et.al.,Towards Real-Time Dynamic Spectrum Auctions by Gandhi

13. *Game Theory, Alive.* Anna R. Karlin and Yuval Peres. (Sep 25, 2016 version)

T.Y.B.Sc. (Mathematics) SEMESTER VI	
Practical approach to Game Theory	**Course code: 20US6MTGMT6**

Title	Practical approach to Game Theory	
Course code	20US6MTGMT6	
Credits	2.5	
Module Number	Title	Number of Lectures
Module I	Solving Graphically	10
Module II	Finding Maximin and minimax value of a Payoff matrix and finding saddle points, Dominance principle	18
Module III	Solving as an LPP, Algebraic methods	18

(Practical oriented course. Evaluation will be 40% internal having Projects and 60% semester end Practical examination of 2 hours duration).

Objective: *Learning different methods to solve some Zero-sum probles.*

Learning outcome: *At the end of the course a student is expected to know how to solve some of the zero-sum kind of problems.*

Module I	Solving Graphically
Module II	Finding Maximin and minimax value of a Payoff matrix and finding saddle points, Dominance principle
Module III	Solving as an LPP, Algebraic methods

T.Y.B.Sc. (Mathematics) Semester VI

Skill Enhancement course (any one)

Preamble

SQL is used to communicate with a database. SQL statements are used to perform tasks such as update data on a database, or retrieve data from a database. Some common relational database management systems that use SQL are: Oracle, Sybase, Microsoft SQL Server, Access, Ingres, etc. SQL stands for Structured Query Language. This version was initially called as SEQUEL (Structured English Query Language) was designed to retrieve and manipulate data stored in IBM's quasi-relational database management systems

Course Objective*: The idea of SQL is common in most tech-friendly businesses, but the reality is that few people have a working understanding of this language.*

By taking an introduction to SQL, participants will be familiar with how SQL works, what it is and how to practically apply it to everyday work at the office.

Learning outcome*: At the end of the course, student will:*

have a broad understanding of database concepts and database management system software

have a high-level understanding of major DBMS components and their function

be able to model an application's data requirements using conceptual modeling tools

be able to write SQL commands to create tables and indexes, insert/update/delete data, and query data in a relational DBMS.

R Programming

Preamble

R programming is a extensive Catalogue of Mathematical, Statistical and Graphical methods. It is highly useful in the area of Machine learning, Data analysis etc.

Objective*: R Programming is useful tool for students of Mathematics to learn for those interested to pursue Data Science related area.*

Learning outcome*: After learning this course a student should comfortably use R software for various Mathematical and Statistical related problems.*

Course code	**Title**
20US6MTDBL7	SQL and LaTeX
20US6MTRL7	R Programming and LaTeX

Skill Enhancement courses

SEMESTER VI	
Course code 20US6MTDM	**Soft skills-SQL**

Course code	20US6MTSESQL	
Title	Soft skills-SQL	
Credits	2	Teaching load
Module I	Relational Database Management System	4
Module II	SQL commands and functions	16

Practical based course. Evaluation will be continuous evaluation and there will not be any end semester examination

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Module I **Relational Database Management System**

Introduction to Database Concepts: Database, Overview of database management system. Database Languages- Data Definition Language (DDL) and Data Manipulation Languages (DML).

Entity Relation Model: Entity, attributes, keys, relations, Designing ER diagram, integrity constraints over relations, Conversion of ER to relations with and without constraints.

Module II **SQL commands and functions**

Creating and altering tables: CREATE statement with constraints like KEY, CHECK, DEFAULT, ALTER and DROP statement.

Handling data using SQL: selecting data using SELECT statement, FROM clause, WHERE clause, HAVING clause, ORDER BY, GROUP BY, DISTINCT and ALL predicates, Adding data with INSERT statement, changing data with UPDATE statement, removing data with DELETE statement.

Functions: Aggregate functions-AVG, SUM, MIN, MAX and COUNT, Date functions-ADD_MONTHS(), CURRENT_DATE(), LAST_DAY(), MONTHS_BETWEEN(), NEXT_DAY(). String functions- LOWER(), UPPER(), LTRIM(), RTRIM(), TRIM(), INSTR(), RIGHT(), LEFT(), LENGTH(), SUBSTR(). Numeric functions: ABS(), EXP(), LOG(), SQRT(), POWER(), SIGN(), ROUND(number).

Advance LaTeX

References:

(a) Database Management Systems, Ramakrishnam, Gehrke, McGraw-Hill

(b) Ivan Bayross, "SQL, PL/SQL -The Programming language of Oracle", B.P.B. Publications, 3rd Revised Edition.

(c) George Koch and Kevin Loney , ORACLE "The Complete Reference", Tata McGraw Hill,New Delhi.

Additional References:

a) Elsmasri and Navathe, "Fundamentals of Database Systems", Pearson Education.

b) Peter Rob and Coronel, "Database Systems, Design, Implementation and Management", Thomson Learning

c) C.J.Date, Longman, "Introduction to database Systems", Pearson Education.

d) Jeffrey D. Ullman, Jennifer Widom, "A First Course in Database Systems", Pearson Education.

e) Martin Gruber, "Understanding SQL",B.P.B. Publications.

f) Michael Abbey, Michael J. Corey, Ian Abramson, Oracle 8i – A Beginner's Guide, Tata McGraw-Hill.

Soft skills-R Programming and LaTeX	Course code: 20US5MTSR

(This Course is Practical oriented course. Evaluation will be continuous evaluation and there will not be any end semester examination).

Course code	20US6MTSER	
Title	Soft skills-R	
Credits	2	Teaching load
Module I	R Programming	12
Module II	Simulation and code profiling	12

Module I	Introduction to R
	Overview of R, R data types and objects, reading and writing data
	Control Structures, functions, scoping rules, Date and Time
Module II	Loop functions, debugging tools, Simulation and code profiling
Module III	Advance LaTeX

.References:

1. R for beginners by Sandip Rakshit McGraw Hill
2.Beginning R by Dr. Mark Gardener
3. R for Everyone by Jared P. LanderAddison Wesley Data And Analytic Series.

Evaluation Pattern:

Internal Assessment:

For each of the Core theory courses and Discipline specific course I (DSE-I) there will multiple test and activity-based assessment. The tests will involve objective and descriptive kind. It can involve a small project. Some of the projects maybe a group project. Part of the internal evaluation can also be undertaken by students by enrolling themselves in relevant online courses developed by NPTEL/SWAYAM etc. Such courses will require to be minimum 2 to 3 weeks duration.

DSE_II is practical oriented course and 40% of the evaluation will be based on Projects and 60% will be semester end practical examination of 2 hours duration.

Skill Enhancement course is a practical based course and will be evaluated on a continuous basis. There will not be any end semester examination.

Project evaluation will be based on the following criterion:

Content 60% (relevance, introduction, pre-requisites, application and what further treatment can be given)

Presentation (of the content, Photos / diagrams and graphs attached, PPT) 30%

Viva-voce 10%

Semester end examination:

In each Theory course there will be one question from each module. All question will be according to the level of the module as per the weightage given (number of Lectures allotted for that Unit). There will be internal option.

Practical evaluation:

At the end of the semester there will a practical examination for all core courses and DSE-II courses (other than Skill Enhancement course). The Practical exam in each course will be conducted in different days, each of 2 hours duration.

Practical Question Paper pattern for each course (core courses will be as follows (for nonprogramming courses):

Q 1. Multiple choice questions. Out of 12, a student will need to solve 8. Maximum marks allotted for this will be 16 marks. Number of questions from a module will proportionate to the level of the module.

Q2. Will be descriptive type questions and will carry maximum of 24 marks. A student will attempt any three out of four questions. At least One question from each module

DSE-II (60% part, semester end evaluation))

For DSE-II Programming courses (60%-part, semester end evaluation) there will be Three questions. Question 1 and 2 will be that of program writing and execution and Q3 will be debugging of a prewritten program or writing algorithm or converting algorithm to program.

Each of the DSE-II course will be evaluated internally for 40% as projects.

Certified Journal and viva-voce (by internal examiner) will carry maximum of 10 marks