K. J. SOMAIYA COLLEGE OF SCIENCE AND COMMERCE

AUTONOMOUS – Affiliated to University of Mumbai

Re-accredited "A' Grade by NAAC
Vidyanagar, Vidyavihar, Mumbai 400 077

Syllabus of

Programme: B.Sc.

SYBSc
Semester III and IV
Mathematics Syllabus
Implementation year 2019-2020

Semester III											
Course No	Course Title	Course code	Credits	Hours	Periods (50 min)	Unit/ Module	Lectures (50 minutes) per module (app)	**Examination**		
								Internal Marks	External Marks	Total Marks
THEORY: Core courses										
I	**CALCULUS-II**	**19US3MT1**	2	37.5	45	4	11	40	60	100
II	**Linear Algebra I**	**19US3MT2**	2	37.5	45	4	11	40	60	100
III	**Combinatorics**	**19US3MT3**	2	37.5	45	4	11	40	60	100
Practical on Core Courses										
I	**Calculus, Linear Algebra and Combinatorics**	**19US3MTP**	3	60	72	2	20	___	_	100
TOTAL			9					280	420	1000

Semester IV										
Course No	Course Title	Course code	Credits	Hours	Periods (50 min)	Unit/ Module	Lectures (50 minutes) per module (app)	**Examination**		
								Internal Marks	External Marks	Total Marks
THEORY: Core courses										
I	**Calculus III**	**19US4MT1**	2	37.5	45	4	11	40	60	100
II	**Linear Algebra II**	**19US4MT2**	2	37.5	45	4	11	40	60	100
III	**Ordinary Differential equations and Numerical Methods-Financial Mathematics**	**19US4MT3**	2	37.5	45	4	11	40	60	100
Practical on Core Courses										
I	**Mathematics practical**	**19US4MTP**	3	60	72	2	20	___	_	100
TOTAL			9					280	420	1000

Preamble

Mathematics is universally accepted as the queen of all sciences. This fact has been confirmed with the advances made in Science and Technology. Mathematics has become an imperative prerequisite for all the branches of science such as Physics, Computer Science etc. The syllabus in Mathematics for Semester III and Semester IV of the B.Sc. Programme aims at catering to the needs of the students of all these branches and also learns core areas of Mathematics.
In each semester a student will learn three Mathematics courses, of which the first two courses are core and the third is computational in nature dealing with Application oriented courses.
Course I in Semester III is **CALCULUS II** which is in continuation of calculus I, learnt in semester II of F.Y.B.Sc.
The course II is **LINEAR ALGEBRA I** in semester III and **LINEAR ALGEBRA II** in semester IV.
In the course III of semester III, a student has an option to either study Graph theory or Cryptography. Accordingly, the name of the course will be either **COMBINATORICS** (those who opt for Graph theory) and **COMBINATORICS AND CRYPTOGRAPHY** (for those who opt for Cryptography instead of Graph theory)
In Semester IV a student has an option to either learn Second Ordered Ordinary Differential Equations and how numerical methods can be applied to problems in Algebra and Calculus or the student will learn Second Ordered Ordinary Differential Equations and how numerical methods can be applied to problems in Algebra and will explore basics of financial mathematics. Accordingly, the name of the course will be '**SECOND ORDERED ORDINARY DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS**' (for those who decides to learn Numerical methods extensively) and '**SECOND ORDERED ORDINARY DIFFERENTIAL EQUATIONS, NUMERICAL METHODS AND FINANCIAL MATHEMATICS**' (for those who decides to explore financial mathematics.

Learning Objectives:
Semester III-Course I: CALCULUS-II
Semester IV-Course I:
RIEMANN INTEGRATION AND INTRODUCTION TO COMPLEX ANALYSIS
Learning the concepts of sequences and series and understanding its convergence/non-convergence. Learning various types of sequences such as monotonic sequences, bounded sequences, sub sequences and Cauchy sequences along with their properties. Understanding the convergence of series via various tests of convergence. Learning continuity of functions in a new light using sequential criterion. Understanding the concept of uniform continuity. The student is also expected to understand the concept of integration through upper and lower Riemann sums. Besides this, the student learns applications of the same. The student gets a motivation towards complex analysis in module 4.
Semester III-Course II: LINEAR ALGEBRA-I

Semester IV-Course II: LINEAR ALGEBRA-II

As system of linear equations is inevitable in any topic of linear algebra, emphasis is given on solving a system and geometrically interpreting it. Understanding these concepts and applying them in various cases is one of the objectives of the course. Understanding linear algebra through geometry is one of the aim of the course. Learning concepts of vector spaces, finite dimensional vector space and subspace over Reals, extending a linearly independent set to form a basis, Inner product space and obtaining orthonormal basis using Gram-Schmidt process.

a student will learn Linear transformation, Matrix associated with the linear transformation and evaluate the eigen values and corresponding eigen vectors and applying them in various situations. The course culminates with students understanding diagonalization of a square matrix and some of its application.

Semester III-Course III: COMBINATORICS / COMBINATORIC AND CRYPTOGRAPHY

Semester IV-Course II: SECOND ORDER DIFFEREENTIAL EQUATIONS AND NUMERICAL METHODS / SECOND ORDER DIFFEREENTIAL EQUATIONS, NUMERICAL METHODS AND FINANCIAL MATHEMATICS

Understand various types of graphs. How some of the real-world problems can be converted to a problem of graph theory or the student will learn some of the classical methods to code and decode a message. Applying Counting Principles to enumerate on various types of situations. The students are expected to understand few techniques and apply them. Students will be expected to understand techniques that can be used to make their calculation simpler and effective. Understanding the need to apply numerical methods to solve mathematical problems when conventional methods fail. The students should be learning how to apply methods to solve an equation/system of equations, interpolate values for an available set of data. Differentiate, Integrate and solve differential equation by using some of the numerical methods and analytical methods (second order). Student opting for financial mathematics will be learn and understand among others, the concept of arbitrage, forward contract and options (call and put) and workout problems involving them.

Learning outcomes:

Semester III-Course I: CALCULUS-II

Semester IV-Course I: RIEMANN INTEGRATION AND INTRODUCTION TO COMPLEX ANALYSIS

At the end of the course, the student will be able to analyse the convergence of sequences and series. This enables the student to perceive continuity from a different angle. The student should be well equipped to understand continuity of complex valued functions. This facilitates the study of uniform convergence of sequences and series of functions in later semesters. The student should be able to understand the nuances of integration which will pave the way towards double and triple integration.

Semester III-Course II: LINEAR ALGEBRA-I

Semester IV-Course II: LINEAR ALGEBRA-II

Students will be able to find solution of a system of linear equations. Students will be able to find if the given set forms a vector space. Students will be able to find basis and dimension of a vector space. Students will be able to check the geometrical properties for given vectors in inner product space. Students will be able to find matrices associated with a linear transformation. Students will be able to find eigen values and eigenvectors of a matrix. Student will be able to diagonalize a suitable matrix. Student will be able to use diagonalization of a matrix to find its power.

Semester III-Course III: COMBINATORICS / COMBINATORICS AND CRYPTOGRAPHY
Semester IV-Course II: SECOND ORDER DIFFEREENTIAL EQUATIONS AND NUMERICAL METHODS / SECOND ORDER DIFFEREENTIAL EQUATIONS, NUMERICAL METHODS AND FINANCIAL MATHEMATICS

Use definitions and theorems to formulate graph theoretical models to solve some of the 'real-world' problems or encode or decode a cryptic message using some of the classical methods of cryptography. They should be able to analyse combinatorial objects satisfying certain properties and enumerate. They will be in a position to apply methods that can quickly calculate when traditional methods would have been tedious and time consuming. The students will be in a position to apply some of the analytical methods to solve second order differential equation. Student opting for financial mathematics will be able to understand the concept of arbitrage, forward contract and options (call and put) and workout problems involving such concepts.

SYBSc Course I Semester III
Course Code: 19US3MT1
CALCULUS-II

Module 1	Sequences of real numbers and their convergence	(12 Lectures)
Module 2	Cauchy Sequences over R.	(12 lectures)
Module 3	Series of real numbers	(12 Lectures)
Module 4	Continuity and Uniform Continuity over R	(12 Lectures)

Syllabus

Module 1	**Sequences of real numbers and their convergence**
1.1	Sequence of real numbers, its range set. Boundedness of a sequence of real numbers in terms of its range. Monotonic sequences.
1.2	Definition of a convergent sequence in terms of ε and n_0. Uniqueness of limit of a convergent sequence. Boundedness of convergent sequence.
1.3	Algebra of convergent sequences. Convergence of a monotonic and bounded sequence.
1.4	Application of Nested Intervals property: If is a nested sequence of closed and bounded intervals with as n → ∞ then is a singleton. Application of Bolzano Weierstrass Theorem for Sequences (Both statements only. The student will prove them as a part of a project)
Module 2	**Cauchy Sequences over R.**
2.1	Subsequence of a sequence. Convergence of a sequence implies convergence of its subsequence but not conversely. Convergence of $(x_{2n}), (x_{2n-1})$ to the same limit p implies convergence of (x_n) to p. Every bounded sequence has a convergent subsequence.
2.2	Limit superior and Limit inferior of a sequence.
2.3	Definition of a Cauchy sequence.
2.4	Every convergent sequence is Cauchy. Every Cauchy sequence is bounded. If a subsequence of a Cauchy sequence is convergent then the sequence itself is convergent. Every Cauchy sequence of real numbers is convergent. Completeness of R.
Module 3	**Series of real numbers**
3.1	Series of real numbers. Terms of a series and partial sums. Summability / Convergence of a real series in terms of convergence of its partial sums.
3.2	Convergence of series implies convergence of n^{th} term to zero, and converse is false. Simple examples of convergent series and divergent series without involving tests. Sum of two series and multiplication of each term of a series by a

	scalar.
3.3	Geometric series and p-series. P-series converges if and only if p>1. (without proof) Illustration for p=1, p=2. Series of non-negative terms and Comparison test (simple form).Alternating series and Leibnitz test.
3.4	Absolute convergence and conditional convergence. Absolute convergence implies conditional convergence and converse is false.
3.5	Ratio test and root test (without proof) and problems based on these tests. Power Series. Calculation of radius of convergence and interval of convergence.
Module 4	**Continuity and Uniform Continuity over R**
4.1	Review of continuous functions.
4.2	Sequential criterion for continuity and its equivalence with the $\varepsilon - \delta$ definition. Continuous image of a Cauchy sequence need not be Cauchy.
4.3	Uniformly continuous functions. Monotonic function defined on a closed and bounded interval is uniformly continuous. Uniformly continuous image of a Cauchy sequence is Cauchy.
4.4	Proof of Intermediate Value Property. Proof of maximum value property of continuous functions on a closed and bounded interval. Continuous function on a closed and bounded interval is uniformly continuous.

Reference books:

1. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
2. R.G. Bartle and D. R Sherbert, *Introduction to Real Analysis,* John Wiley and Sons (Asia) P.Ltd., 2000.
3. G.B. Thomas and R.L. Finney, *Calculus*, Pearson Education, 2007.

Additional Reference books:

1. H. Anton, I. Bivens and S. Davis, *Calculus*, John Wiley and Sons, Inc., 2002.
2. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
3. T. M. Apostol, *Calculus* (Vol. I), John Wiley and Sons (Asia) P. Ltd., 2002.
4. W. Rudin, Principles of mathematical Analysis, Tata McGraw- Hill Education in 2013.

SYBSc Paper II Semester III
Course Code: 19US3MT2
Linear Algebra-I

Module I- Matrices and system of equations (12 lectures)
Module II Vector Spaces (12 lectures)
Module III Basis and Dimension (12 lectures)
Module IV Inner Product spaces (12 lectures)

Syllabus

Module I	**Matrices and system of equations**
1.1	Matrices over R and C. special types of matrices like diagonal matrix, zero matrix, lower and upper triangular matrix, symmetric matrix etc. Algebraic operations on matrices and their properties. System of homogeneous and non- homogeneous linear equations.
1.2	Existence of non-trivial solution for homogeneous system for m<n. Row echelon form of a matrix, Elementary row operations, equivalent systems, Solutions of m homogeneous equations in n unknowns by
1.3	Gauss elimination and their geometric interpretation.
Module 2	**Vector Spaces**
2.1	Definition of vector space, simple examples.
2.2	Subspaces, sum and intersection of subspaces, direct sum of vector space,
2.3	linear combination of vectors, linear span of a subset of a vector space. Simple examples
Module 3	**Basis and Dimension**
3.1	Linear dependence and independence, Generating set, Basis and Dimensions of a Vector Space. Examples of some infinite dimensional vector spaces like C[a,b] and polynomial space.
3.2	Basis in terms of a maximal linearly independent set and as minimal generating set. Results like extension/reduction of a given set to a basis. Row space and Column Space of a matrix, Row rank and Column rank
3.3	and their equivalence. Computing rank of a matrix by row reduction.
Module 4	**Inner Product spaces**
4.1	Dot product in Euclidean space. General inner product spaces. Norm induced by inner product, Cauchy Schwarz inequality. Angle between
4.2	two vectors, Orthogonality, Pythagoras theorem, triangle inequality, parallelogram law and similar identities, Orthogonal projection onto a line,
4.3	

Orthogonal and orthonormal sets. Orthonormal. basis, Gram-Schmidt Orthogonalization, Orthogonal complement, results related to orthonormal basis.

Reference books:

(1) S. Kumareson -Linear algebra : a geometric approach
(2) Serge Lang – Linear algebra
(3) I.K.Rana – Linear algebra
(4) Schaum'series – Linear algebra
(5) Linear Algebra –K Hoffman and R Kunze
(6) Introduction to Linear Algebra –Gilbert Strang

SYBSc Paper III Semester III
Course Code: 19US3MT3
COMBINATORICS/ CRYPTOGRAPHY AND COMBINATORICS

Module 1	Basic Counting (15 lectures)
Module 2	Advanced Counting (15 lectures)
Module 3	Graph theory (15 Lectures) Or Cryptography (15 Lectures)

Syllabus

Module 1	**Basic Counting**
1.1	Addition and multiplication principles.
1.2	Counting sets of pairs, two-way counting
1.3	Distribution of objects, multinomial numbers, combinatorial interpretation of multinomial theorem.
1.4	Derangements on n symbols, d_n. Arithmetic applications including Euler- φ function
1.5	Principle of Inclusion and exclusion and applications
Module 2	**Advanced Counting**
2.1	Proving the identities using combinatorial arguments such as . . .

i) Vandermonde's identity

$$\sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$$

ii) Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

iii)

$$\sum_{i=0}^{k}\binom{k}{i}^2 = \binom{2k}{k}$$

2.2	Pigeon hole principle and its applications.
2.3	Recurrence relation, definition of homogeneous, non-homogeneous, linear and non-linear recurrence relation, obtaining recurrence relation in counting problems, solving homogeneous and non-homogenous recurrence relation by using iterative method, solving a homogeneous recurrence relation of second degree using algebraic method
Module G3	**Graph theory**
G3.1	Definition of simple and multi-graph, Types of graph such as complete graph, regular graph, bipartite graph, Complement of a graph, Self-complementary graph. Subgraphs. Trees, subtrees, Spanning tree of a graph.

G3.2	Degree of a vertex, Adjacency matrix, incidence matrix, walks, trails, path, circuit, cycle, connected graph, Component, Eulerian graphs, isomorphism of graphs. Bridges and Cut vertex. Shortest path problem: Dijkstra's algorithm.
G3.3	Rooted tree, m-ary tree, Kruskal's algorithm for minimal weighted spanning tree.
G3.4	Simple properties of graphs Such as Hand shaking lemma. G or its complement is always connected. A self-complementary graph should have 4n or 4n+1 number of vertices. A simple graph with at least 2 vertices has at least two vertices with the same degree. A simple graph on p vertices and p-1 number of edges has either a pendent or an isolated vertex. A connected graph T is a tree iff every edge is a bridge. A tree on n vertices has n- 1 number of edges. Calculating number of vertices of degree 1, m+1 in a complete m-ary tree.
Module C3	Cryptography (as an option to Graph theory)
C3.1	Review Congruences, Euler phi function, Solving Diophantine equation, congruence equation, application of Chinese remainder theorem, primality testing, Fermat primality testing, Miller-Rabin Primality test.
C3.2	Basic notions such as encryption (enciphering) and decryption (deciphering). Cryptosystems, symmetric key cryptography. Simple examples such as shift cipher, affine cipher, hill's cipher. Vigenere cipher, substitution cipher,
C3.3	Playfair and ADFGX cipher, block cipher, one-time pad,
C3.4	Concept of Public Key Cryptosystem; RSA Algorithm.

Reference books:

- Norman Biggs: Discrete Mathematics, Oxford University Press.
- Kenneth Rosen; Discrete Mathematics and Its Applications (SIE); MacGraw Hill.
- Schaum's outline series : Discrete mathematics

More Reference books:

For Module 1 and 2:

1. Richard Brualdi: Introductory Combinatorics, John Wiley and sons.
2. V. Krishnamurthy: Combinatorics – Theory and Applications, Affiliated East West Press.
3. Applied Combinatorics: Allen Tucker, John Wiley and Sons.

For Module G3:

Frank Harary; Graph theory

Deo, Narsingh; Graph Theory with Applications to Engineering and Computer Science; Eastern Economy edition; PHI

Bondy and Murty; Graph theory; Springer
Reinhard Diestel; Graph theory; Springer
Introduction to Graph theory; Douglas B. West; Pearson Education.

For Module C3:

1. Introduction to Cryptography with Coding Theory; Wade Trappe and Lawrence C. Washington; Pearsons Education International
2. Notes on Cryptography; Peter Cameron.
3. Understanding Cryptography; Christof Paar AndJan Palzl; Springer

SYBSc Semester III
Course code: 19US3MTP
Practical Course

Suggested Practical:

Module 1.

1 Problems on finding terms of a sequence, checking bounded/unbounded sequences and monotonic sequences.
2 Convergence of a sequence in terms of ε and n_0 and subsequence.
3 Limit superior and limit inferior of a sequence and problems on Cauchy sequences.
4 Convergence of a series using Comparison test (simple form) and Leibnitz test.
5 Convergence of a series using Ratio test and root test, and computing radius of convergence of power series.
6 Continuity of functions using sequential criterion and problems on uniform continuous functions.

Module 2:

1 Verification of various properties of matrices, geometrical interpretation of system of linear equations and its solutions.
2 Solving a system of linear equations by Gauss Elimination method.
3 Examples related to vector spaces. Examples of finding subspaces, linear span of given set.
4 Finding whether the given set is linearly dependent or independent, extending or reducing the given set to a basis.
5 Finding row space and column space of given matrix, verifying equivalence of row rank and column rank. Finding rank of a matrix by row reduction.
6 Examples on inner product spaces, finding norm induced by it.

Finding if the set is Orthogonal, Gram Schmidt orthogonalization

process.

Module 3:

G1. Drawing Graphs and obtaining the matrix representation of the graphs. Finding the walks and path, Triangles in a graph. Complement of the graph.

G2. Isomorphism of graphs. Eulerian graph. Connected graph, Dijkstra's and Kruskal's algorithm.

3. Problems based on counting principles, Two-way counting. Multinomial theorem, identities, permutation and combination of multi-set.
4. Inclusion-Exclusion principle, Euler phi function. Derangement
5. Pigeon hole principle. Combinatorial reasoning-based problems.
6. Recurrence relation.

In place G1 and G2 Students opting for Cryptography will do the following Two practical

C1 Problems based on application of Chinese remainder theorem, primality testing, Fermat primality testing, Miller-Rabin Primality test. Shift cipher, affine cipher, hill's cipher. Vigenere cipher, substitution cipher.

C2 Playfair and ADFGX cipher, block cipher, one-time pad, RSA Algorithm.

SYBSc Paper I Semester IV

Course Code: 19US4MT1

Riemann Integration and Introduction to Complex Analysis

Module 1	Riemann Integration: definition and examples	(12 Lectures)
Module 2	Riemann Integration: Riemann criteria and Fundamental theorems of Calculus.	(12 Lectures)
Module 3	Applications of Riemann integrals	(12 Lectures)
Module 4	Introduction to complex Analysis	(12 Lectures)

Syllabus

Module 1	**Riemann Integration: definition and examples**
1.1	Partition of a set, partition of an interval in a finite number of subintervals.
1.2	Upper Riemann sum and lower Riemann sum of a function with respect to a partition.
1.3	Upper integral, lower integral of a function. Definition of Riemann integrability and integral of a function over an interval. Simple examples.
Module 2	**Riemann Integration: Riemann criteria and Fundamental theorems of Calculus.**
2.1	Riemann criteria for integrability with examples.
2.4	Basic properties of R-integrable functions.
2.3	Monotonic bounded functions over a bounded interval are R-integrable. Continuous functions defined over a closed and bounded interval are R-integrable. R- integrability of piecewise continuous functions over bounded intervals.
2.4	Fundamental theorems of calculus and applications .
Module 3	**Applications of Riemann integrals**
3.1	Mean Value theorem for integrals. Change of variable formula.
3.2	Computation of area under a curve, area of bounded regions.
3.3	Volume of regions obtained by rotating a curve about an axis.
3.4	Improper integrals and their convergence.
Module 4	**Introduction to complex Analysis**
4.1	Review of Algebraic properties of complex numbers.
4.2	Metric topology of Complex Numbers : Neighbourhood of a point . Open sets and closed sets. Set of complex numbers as a metric space with distance given by $d(z, w) = \|z - w\|$ Notion of open region in **C**.
4.3	Sequences and series of complex numbers. converges in C if and only if both converge in R. Completeness of C.
4.4	Functions from a metric space to a metric space. Limit of a function. Continuity of a function in terms of limit. Expressing a function $f:\Omega\rightarrow C$

	as $f(x+yi)=u(x,y)+v(x,y)i$ where $u,v:\Omega \to R$ considering Ω as a subset of R^2. A function is continuous at $a+bi$ and only if u,v are continuous at (a,b)
4.5	Sum rule, product rule, division rule for finding limits of a complex valued function of one complex variable.
4.6	Understanding a complex valued function of a complex variable graphically by plotting images of straight lines and curves. Examples such as $\to z^2, z \to \bar{z}$. Definition of the exponential function . Periodicity of the function. Definitions of trigonometric and hyperbolic functions in terms of the exponential function.

Recommended Books for Modules 1,2 and 3:

1. J. E. Marsden, A. J. Tromba and A. Weinstein, Basic multivariable calculus.
2. R.G. Bartle and D. R Sherbert, *Introduction to Real Analysis,* John Wiley and Sons (Asia) P.Ltd., 2000.
3. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

Recommended Book for Modules 4:

J. W. Brown and R. V. Churchill, Complex variables and applications, McGraw-Hill International, sixth edition.

Additional Reference books:

1. H. Anton, I. Bivens and S. Davis, *Calculus*, John Wiley and Sons, Inc., 2002.
2. T. M. Apostol, *Calculus* (Vol. I), John Wiley and Sons (Asia) P. Ltd., 2002.
2. W. Rudin, Principles of mathematical Analysis, Tata McGraw- Hill Education in 2013.

SYBSc Paper II Semester IV

Course Code: 19US4MT2

Linear Algebra-II

Module 1: Linear Transformations (12 Lectures)
Module 2: Matrix Representation of a linear transformation (12 Lectures)
Module 3: Eigen values and Eigen vectors (12 Lectures)
Module 4: Diagonalization (12 Lectures)

Syllabus

Module 1:	**Linear Transformations**
1.1	Definition of Linear Transformation, examples. properties that follow consequently from definition,
1.2	Determining a linear transformation by its values on a basis. Kernel and Image of a Linear transformation, Rank-Nullity theorem, composite of a linear transformation,
1.3	Non-singular linear transformation, Linear Isomorphism, related results.
Module 2:	**Matrix Representation of a linear transformation**
2.1	Representation of a linear transformation by a matrix, matrix of sum, scalar multiple, inverse and composite of linear transformation.
2.2	Equivalence of rank of a matrix and a linear transformation associated with it.
2.3	The solutions of non-homogeneous system of linear equations represented by AX=B.
Module 3:	**Orthogonal Transformations and isometries**
3.1	Orthogonal transformations definition and simple examples, isometry of a real finite dimensional inner product space. Translations and reflections with respect to a hyper plane.
3.2 3.3	Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space. Characterization of isometries as composite of orthogonal transformations and isometries.
3.4	Orthogonal transformation of $\mathbb{R}^2$. Any orthogonal transformation in $\mathbb{R}^2$ is a reflection or a rotation.
Module 4:	**Quotient spaces:**

4.1 4.2	Definition of Coset for a real vector space V and a subspace W, the quotient space V/W. First Isomorphism theorem of real vector spaces (Fundamental theorem of homomorphism of vector spaces.)
4.3	Dimension and basis of the quotient space V/W, when V is finite dimensional.

Reference books:

1. S. Kumareson -Linear algebra : a geometric approach
2. Serge Lang – Linear algebra
3 I.K.Rana – Linear algebra
4 Schaum'series – Linear algebra
5 Linear Algebra –K Hoffman and R Kunze
6 Introduction to Linear Algebra –Gilbert Strang

SYBSc Paper III Semester IV
Course Code: 19US4MT3
Second order Ordinary Differential Equations and Numerical Methods/ Second order Ordinary Differential Equations Numerical Methods and Financial Mathematics

Module 1	Second order Linear Differential equations (15 Lectures)
Module 2	Numerical methods-I (15 Lectures)
Module N3	Numerical methods –II (15Lectures) OR
Module F3	Introduction to Financial Mathematics

Syllabus

Module 1	**Second order Linear Differential equations**
1.1	Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.
1.2	The homogeneous equation with constant coefficients. auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.
1.3	Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.
Module 2	**Numerical methods-I**
2.1	Errors in Numerical calculations: Inherent errors: Round off errors, Truncating errors. Absolute errors; relative errors and percentage errors. Control of Numerical Errors.
2.2	Numerical methods to solve an equation: Newton Raphson method. With derivation and rate of convergence. Muller's method.
2.3	Solving system of equations: LU factorization: Do-Little method Gauss Seidel iterative method
2.4	Interpolation: Newton Gregory forward interpolation Lagrange's interpolation
Module N3	**Numerical methods –II**
N3.1	Numerical differentiation: Fourth order central-difference Differentiation formulas with derivation.

	Derivatives of unequally spaced Data with derivation.
N3.2	Numerical Integration:
	Simpson $\frac{1}{3}$rd rule and $\frac{3}{8}$rule with error analysis.
N3.3	Numerical Solution to Ordinary Differential Equations:
	Taylors method
	Runga Kutta's fourth order method
Module F3	**Introduction to Financial Mathematics**
F3.1	**Simple market model**
	(i) Basic Notions & Assumptions
	(ii) No Arbitrage Principle
	(iii) One step Binomial Model
	(iv) Risk & Return
	(v)Idea of Forward Contracts, Call & Put options
F3.2	(vi) Managing Risk with options
	Risk- Free Assets
F3.3	(i) Time value of Money
	(ii) Money Market
F3.4	
	Risky Assets
	(i) Dynamics of stock prices
	(ii) Binomial Tree Model & Risk Neutral Probability
	Applications of No-Arbitrage Principle
	(i) To the Binomial Tree Model
	(ii) To Pricing Forward Contracts
	(iii) To Put-Call Parity in Options

References:

Numerical Methods:

1) Numerical methods by E. Balaguruswamy, *Tata McGraw Hill*
2) Introductory methods of Numerical Analysis. By S. S. Sastry.
3) Numerical Methods for engineers. By Steven C. Chapra and Raymond Canale; Fifth Edition; *Tata McGraw hill education private ltd*.:

Financial Mathematics:

1) Mathematics for Finance An Introduction to Financial Engineering Marek Capinski and Tomasz Zastawniak *Springer Undergraduate edition -2003(Freely downloadable)*
2) Hull and Basu- Options, Futures and other derivatives
3) Paul Wilmott – Quantitative Finance

Ordinary differential Equations:

1. G. F. Simmons, Differential equations with applications and historical notes, McGraw Hill.
2. E. A. Coddington, An introduction to ordinary differential equations, Dover Books.
3) Shepley L. Ross; Differential Equations, 3ed Paperback – 2007
4) Morris Tenenbaum, Harry Pollard; Ordinary Differential Equations (Dover Books on Mathematics) Revised ed. Edition.

SYBSc Semester IV
Course code: 19US4MTP
Practical Course

Practical:

Module 1:

1 Problems on Partitions and refinements and calculation of upper sum, lower sum and calculation of upper integral, lower integral and Riemann integral using the definition.
2 Problems on Riemann integrable functions using Riemann Criterion and problems on fundamental theorem of calculus.
3 Mean value theorem, Change of Variables Formula, computation of area under a curve, area of bounded regions and volume of regions obtained by rotating a curve about an axis.
4 Convergence of improper integrals, applications of comparison tests.
5 Review of algebraic and geometrical properties of complex numbers and interpretation of inequalities and equations as regions and curves in **C**.
6 Sequences and series of complex numbers, limit and continuity of functions of complex variables, image of curves and regions under action of complex valued functions.

Module 2:

1 Problems on Linear transformations, finding a L.T. with given conditions.
2 Verifying rank nullity theorem, determining isomorphism between given pair of vector spaces.
3 Finding matrix associated with given linear transformation and vice versa, verification of the relationship of algebraic properties of matrix and linear transformation.
4 Solving a system of linear equations completely, finding the dimension of solution set of underlying homogeneous system. Finding Eigen values and Eigen vectors of square matrices.
5 Identifying given orthogonal transformation as rotation or reflection from its matrix. Simple problems on isometries.
6 Problems on quotient spaces, application of fundamental theorem to solve simple problems.

Module 3:

1		Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
2		Solving equations using method of undetermined coefficients and method of variation of parameters. Solving second order linear ODEs.
3	a)	Solving an equation.
3	b)	Solving a system of equations.
4		Interpolations.
N5		Numerical differentiation and integration.
N6		Numerical solution to ODE.
F5		Risk free investment and Problems involving Arbitrage
F6		Forward contract and Options

Evaluation Pattern:

Internal Assessment:

For each of the theory course there will 3 tests of 15 marks *(subject to change). From these three tests, best two will be considered. The tests will involve objective and descriptive kind. It can involve a small project. Some of the projects maybe a group project. The 30 marks internal evaluation can also be undertaken by students by enrolling themselves in relevant online courses developed by NPTEL/SWAYAM etc. Such courses will require to be minimum 2 to 3 weeks duration for a 30 marks evaluation.

There will be a 10 marks evaluation based on various activities organised by students/teachers.

Semester end examination:

In each course there will be one question from each module. All question will be according to the level of the module. There will be internal option.

Practical evaluation:

At the end of the semester there will a practical examination. The Practical exam in each module will be conducted in different days, each of 2 hours duration.

Paper pattern for each module will be as follows:

Q 1. Multiple choice questions. Out of 12, a student will need to solve 8. Maximum marks allotted for this will be 16 marks.

Q2. Will be descriptive type questions and will carry maximum of 24 marks. A student will attempt any three out of four questions.

Certified Journal will carry maximum of 10 marks