K. J. SOMAIYA COLLEGE OF SCIENCE AND COMMERCE

AUTONOMOUS – Affiliated to University of Mumbai

Re-accredited "A' Grade by NAAC
Vidyanagar, Vidyavihar, Mumbai 400 077

Syllabus of

Programme: B.Sc.

FYBSC

MATHEMATICS

SEMESTER I AND II

Implementation Year 2018-19

Semester I										
Course No	Course Title	Course code	Credits	Hours	Periods (50 min)	Unit/Module	Lectures (50 minutes) per module (app)	**Examination**		
								Internal Marks	External Marks	Total Marks
THEORY: Core courses										
I	**Ordinary Differential equation and Real Number System**	**18US1MT1**	2	37.5	45	4	11	40	60	100
II	**Discrete Mathematics**	**18US1MT2**	2	37.5	45	4	11	40	60	100
Practical on Core Courses										
I	**Real Analysis and Discrete mathematics**	**18US1MTP**	2	60	72	2	20	___	_	100
TOTAL			6					280	420	1000

Semester II										
Course No	Course Title	Course code	Credits	Hours	Periods (50 min)	Unit/ Module	Lectures (50 minutes) per module (app)	**Examination**		
								Internal Marks	External Marks	Total Marks
THEORY: Core courses										
I	**Calculus I**	**18US2MT1**	2	37.5	45	4	11	40	60	100
II	**Algebra I**	**18US2MT2**	2	37.5	45	4	11	40	60	100
Practical on Core Courses										
I	**Calculus and Algebra**	**18US2MTP**	2	60	72	2	20	___	_	100
TOTAL			6					280	420	1000

Preamble

Mathematics is universally accepted as the queen of all sciences. This fact has been confirmed with the advances made in Science and Technology. Mathematics has become an imperative prerequisite for all the branches of science such as Physics, Statistics, and Computer Science etc. This proposed revised syllabus in Mathematics for Semester I and Semester II of the B.Sc. Program aims at catering to the needs of the students of all these branches.

The student learns two courses in each Semester wherein Course I deals with "Properties of real Numbers and Calculus" and Course II deals with "Discrete Mathematics."

Course I: Students who have studied mathematics up to H.S.C. exam are exposed to calculus of one variable. Proposed syllabus aims at learning the definitions more thoroughly and knowing several other applications of derivatives and differential equations. This is not going to offer a very hard, rigorous course in calculus, but it will prepare them for a more serious study. Adequate emphases is given on observing graphs of functions and thereby visualize properties of functions of one variable, the reasons behind these properties are then explained through definitions and results. This course will provide a solid background for a student who wishes to learn deeper ideas in mathematics as well as for a student who wishes to pursue study of other sciences where he/she will be able to easily pick up ideas involved with mathematical applications in these sciences.

Course II: In this course Students will learn in the first semester some of the discrete structures in Mathematics and will evolve various tools necessary to understand them. In Semester II a student will study few of the algebraic structures and some of the properties of these structures. The last module (Groups) will be the culmination of all the topics learnt in both the terms.

By the end of the two Semesters a student will be in a position to deal with Differential Calculus of one variable and its application. The students will be equipped with some of the tools needed for understanding higher Mathematics and apply them in other Sciences.

Semester I **Course I** **Course name: Ordinary Differential equation and Real Number System** **Course Code 18US1MT1**

.Module 1 **Ordinary Differential Equations of order 1 and degree 1**

(11 lectures teaching load and 15 lectures students working load) (Maximum marks 12)

1.1 Definition of an ordinary differential equation (ODE).

Degree and order of an ordinary differential equation.

Formulation of a differential equation of order one or more.

1.2 Solving differential equations in the variables separable form.

Conversion of an ODE to the variable separation form by substitution.

Homogeneous differential equations and its solution.

Conversion of a differential equation to homogeneous form by substitution.

1.3 Exact differential equations.

Necessary and sufficient condition for a differential equation to be exact (without proof).

Solution of an exact differential equation.

Integrating factors of a differential equation which is not exact (without proof). Finding such a factor using rules.

Rule 1 If M, N are homogeneous functions of same degree and $Mx + Ny \neq 0$ then $\frac{1}{Mx+Ny}$ is an integrating factor.

Rule 2 If $M = f(xy)y$, $N = g(xy)x$ and $Mx - Ny \neq 0$ then $\frac{1}{Mx-Ny}$ is an integrating factor.

Rule 3 If $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = p(x)$ is a function of x alone then $e^{\int p(x)dx}$ is an integrating factor.

Rule 4 If $\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M} = q(y)$ is a function of y alone then $e^{\int p(y)dy}$ is an integrating factor.

Module 2 Applications of differential equations.

(9 lectures teaching load and 11 lectures students working load) (Maximum marks 12)

2.1 Solving a linear differential equation of order 1 as an application of exact differential equation.

Solving Bernoulli's equation by converting it to a linear differential equation.

2.2 Exponential growth and decay.

Orthogonal trajectories.

Applications to electric circuits.

Module 3 Real Numbers and their basic properties.

(11 lectures teaching load and 14 lectures students working load) (Maximum marks 15)

3.1 $\mathbb{R}$, the set of real numbers with addition and multiplication.

Commutative, associative and distributive properties of the operations of addition and multiplication.

Existence and uniqueness of the additive identity, and the multiplicative identity.

Existence and uniqueness of additive inverse of every real number and multiplicative inverse of every non zero real number.

3.2 Existence of $\mathbb{R}+$, the set of positive real numbers as a maximal subset of real numbers which contains 1 and is closed for addition as well as multiplication.

Order axioms and law of trichotomy. The order relation $\leq$ and its properties w.r.t. addition and multiplication.

3.3 Basic inequalities arising out of non-negativity of square of a real number such as

Cauchy Schwartz Inequality.

Arithmetic and Geometric means of a finite set of real numbers. A.M.≥G.M. inequality.

3.4 Absolute value function and its properties. Triangle inequality. Its consequence: $\left||x|-|y|\right| \leq |x-y|$. Floor function and Ceiling function.

Module 4 Analytic properties of real numbers

(14 lectures teaching load and 20 lectures students working load) (Maximum marks 21)

4.1 Intervals in $\mathbb{R}$. Concept of infinity (∞). Open and closed intervals, bounded and unbounded intervals.

ε-neighborhood of a point and neighborhood of a point in $\mathbb{R}$. Deleted ε-neighborhood of a point. Hausdorff property of $\mathbb{R}$.

4.2 Upper bound and lower bound of a subset of $\mathbb{R}$.

Least upper bound property of $\mathbb{R}$.

Supremum (lub) and infimum (glb) of a non-empty subset of $\mathbb{R}$.

Characterization of lub and glb in terms of ε.

Effect of translation and multiplication of every element by a non-zero scalar in case of a bounded subset of $\mathbb{R}$. Lub and Glb of sum two non-empty bounded subset of $\mathbb{R}$

4.3 Archimedean property of real numbers (With Proof).

Its equivalence with the Density theorem for rational numbers.

Density theorem for irrational numbers.

Projects:

More applications of differential equations

Methods for solving nonlinear differential equations.

Existence and Uniqueness Theorem for linear differential equation of order 1

Indices rules

Application of density theorem

Completeness of $\mathbb{R}$ and representation $\mathbb{R}$ as a number line

References:

1) Differential Equations: With Applications and Historical Notes by G. F. Simmons

2) ROBERT G. BARTLE. DONALD R. SHERBERT. INTRODUCTION TO. REAL. ANALYSIS. THIRD EDITION.

Additional Refrences

1) Ordinary Differential Equations (Dover Books on Mathematics) by M. Tenenbaum

2) Differential equation by G. F. Simmons and S. G. Krantz

3) Methods of Real Analysis by Goldberg

Semester I **Course II** **Course name: Discrete Mathematics** **Course code 18US1MT2**

.Module 1 Sets and relations

(11 lectures teaching load and 15 lectures students working load) (Maximum marks 15)

1.1 Review of:

Union and intersection of sets. Complement of a subset. Power set of a finite set. Distributive properties of sets.

De-Morgan's law.

Difference and symmetric difference of sets.

Cartesian product of sets.

Maximal and minimal subset of a set with a given property.

Zorn's Lemma (only Statement) and its simple applications.

1.2 Definition and examples of relations. Reflexive, symmetric, transitive, equivalence relations. Examples.

Equivalence class and partitions of sets.

Equivalence relations induce partitions and any partition of a set induces an equivalence relation on the set.

1.3 Construction of integers from the set of natural numbers through equivalence classes.

Construction of the set of rational numbers from the set of integers through equivalence classes.

Module 2 Functions

(12 lectures teaching load and 15 lectures students working load) (Maximum marks 15)

2.1 Definition of a function.

Domain, co-domain and the range of a function with examples of special functions such as constant, identity, inclusion, projection, floor and ceiling functions.

Injective, surjective and bijective functions.

2.2 Composition of functions. Composite of bijective function is bijective but converse is not true. If $g\circ f$ is bijective then f is injective and g is surjective.

Invertible functions and the inverse of a function. If f is bijective then its inverse is also a bijective function. Direct and inverse image.

2.3 Binary operations, simple examples.

Finite and infinite sets, cardinality of set, power set.

Countable and uncountable sets

Module 3 Natural numbers and Integers

(11 lectures teaching load and 15 lectures students working load) (Maximum marks 15)

3.1 Well ordering principle. 1 is the smallest natural number. There is no natural number between 1 and 2.

First and second principle of mathematical induction.

Binomial theorem and related identities.

Pascal's law and Pascal's triangle.

3.2 Definition and elementary properties of divisibility in $\mathbb{Z}$.

Division Algorithm.

G.C.D. and L.C.M of two integers and its basic properties including G.C.D. of two integers 'a' and 'b' (not both zero) can be expressed as ma + nb.

Proof of the lemma 'If a = bq + r then GCD(a, b) = (b, r)'. Euclidean Algorithm,

Euclid's Lemma.

Module 4 Prime numbers and linear Diophantine equation

(12 lectures teaching load and 15 lectures students working load) (Maximum marks 15)

4.1 Prime numbers and its basic properties.

Unique Factorization Theorem.

The set of primes is infinite.

The set of primes of the type 4n – 1 and 4n + 1 is infinite.

4.2 Linear Diophantine equation $ax + by = c$

The linear Diophantine equation $ax + by = c$ has solution iff $d \mid c$, where $d = GCD(a, b)$.

If x_0, y_0 is any particular solution then any solution of the given Diophantine equation is given by $x = x_0 + (\frac{b}{d})t$ and $y = y_0 - (\frac{a}{d})t$, for varying t.

Solving simple examples.

Projects

Verification of the group axioms for a non-empty set with respect to the symmetric difference as its operation.

Verification of the Group axioms for a non-empty set with respect to Union as its operation and also intersection as its operation

Sieve of Eratosthenes

Equivalence of the two induction principles

Set of primes of other forms (5n-1 etc) is infinite.

Refrences:

1) Elementary Number theory by David Burton Seventh Edition, McGraw Hill Education (India) Pvt Ltd.

2) Discrete Mathematics by Norman L. Biggs, Clarendon Press.

Additional References:

1) Introduction to theory of numbers by Niven and S. Zuckerman, Wiley Eastern.
2) A survey of Modern Algebra by G. Birkoff and S. Maclane Mac Milan
3) University Algebra by N. S. Gopalkrishnan, New Age International Ltd.

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Semester I

Mathematics Practical Paper

Real Analysis and Discrete Mathematics

1. **Ordinary Differential Equations of order 1 and degree 1**
2. **Applications of differential equations.**
3. **Real Numbers and their basic properties.**
4. **Analytic properties of real numbers**
5. **Sets and relations**
6. **Functions**
7. **Natural numbers and Integers**
8. **Prime numbers and linear Diophantine equation**

Semester II Course I Course name: Calculus I Course code 18US2MT1

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Module 1 Graphs of real valued functions of one variable

(9 lectures teaching load and 10 lectures students working load) (Maximum marks 12)

1.1 Functions of one variable taking real values.

Representation of such a function by its graph.

Observing the increasing, decreasing nature, local extrema and global extrema, concavity of the graph and asymptotes.

1.2 Examples including linear functions, Absolute value function, polynomial function of degree 2 and higher degrees.

Observation that number of roots is less or equal to the degree of the polynomial, observations w.r.t. degree of polynomial and number of extrema, number of points of inflection.

Graphs of trigonometric functions, inverse trigonometric functions, exponential function and logarithmic function. Floor function and ceiling function.

1.3 Characteristic function of a set, Dirichlet's function.

Effect of shifting the origin, composing with reflection, expansion or contraction along X-axis or Y-axis, Observing graph of inverse function as reflection of graph of given function along the line y=x. Observing graph of function $f\left(\frac{1}{x}\right)$ from the graph of $f(x)$.

Graphs of $y = \sin\frac{1,}{x}$, $x\sin\frac{1}{x}$, $x^2\sin\frac{1}{x}$ etc

Module 2 Limits and continuity of real valued functions of one variable

(12 lectures teaching load and 15 lectures students working load) (Maximum marks 15)

2.1 Definition of limit of a function at a point in terms of ε and δ.

Uniqueness of limit.

Boundedness of a function having limit in a neighborhood.

Concept of two sided and one sided limits.

Infinite limit and limit at infinity.

2.2 Sum rule, scalar multiplication, product rule and division rule. Sandwich theorem.

Computations of limits using rules. Nonexistence of limit of functions such as $\sin\frac{1}{x}$

Discussion on the three limits : $\lim\limits_{x\to 0}\frac{\sin x}{x}$, $\lim\limits_{x\to 0}\frac{e^x-1}{x}$, $\lim\limits_{x\to 0}\frac{\log(1+x)}{x}$

2.3 Continuity of a function at a point in terms of limits.

Continuity of a function over an interval, over a set.

Definition of continuity of a function at a point in terms of ε and δ.

Algebra of continuous functions.

Discontinuity of function such as $sin\frac{1}{x}$ at the origin, step function etc

Polynomial functions are continuous.

Function such as $|x|$, $x|x|$, $xsin\frac{1}{x}$, $x^2sin\frac{1}{x}$ etc are continuous

Removable and irremovable discontinuities.

Functions having finite number of discontinuities in an interval.

Functions having infinite number of discontinuities in an interval.

A function which is discontinuous everywhere.

A function which is continuous only at a point.

Composition of continuous functions and taking a limit inside a continuous function.

Composite of continuous functions is continuous but converse is not true.

Two important properties of Continuous functions:

Intermediate value property and Continuous function on a closed and bounded interval attains it maximum value and minimum value.(Without proof)

Module 3 Differentiability of real valued functions of one variable

(12 lectures teaching load and 15 lectures students working load) (Maximum marks 15)

3.1 Definition of differentiability of a real valued function of one variable at a point in terms of a limit.

Differentiability of a function over an interval or a set.

Geometrical interpretation and derivative as a linear approximation in a neighborhood of the point.

Derivative as rate of change and Leibnitz notation.

3.2 Differentiability implies continuity but not converse. Differentiability does not imply continuity over an interval.

Algebra of derivatives.

Chain rule of differentiation.

Computation of derivative of inverse function.

3.3 Rolle's Mean Value Theorem

Lagrange's Mean Value Theorem.

Higher order derivatives and Leibnitz' rule.

Function that are n times differentiable but not differentiable beyond n^{th} order.

Module 4 Applications of derivatives

(12 lectures teaching load and 20 lectures students working load) (Maximum marks 18)

4.1 Taylor's theorem in Lagrange's remainder form (without proof).

Taylor polynomial of n^{th} order and Taylor's series about a point.

Approximation of a n times differentiable function using Taylor's theorem.

4.2 Increasing and decreasing functions.

Local extreme values.

Second derivative test and its extension to test using higher order derivatives (without proof).

L'Hospital's Rule (without proof)

Projects

Taylor's theorem : study of various forms with proof and applications.

Numerical methods to solve equations.

Intermediate value property of continuous functions and its use in numerical methods to solve equations.

Bolzano Weirestrass Theorem

Use of derivatives in curve sketching

Nowhere differentiable but everywhere continuous function.

Transcendence of e, a proof using calculus.

Holder's and Minkowsky inequalities.

Behavior of polynomials a study using calculus.

The Exponential function.

Several ways of proving Cauchy –Schwartz inequality, A.M.≥ G.M.≥H.M., Tchebyshev inequality

Existence and Uniqueness Theorem for linear differential equation of order 1

Rediscovery of mensuration formulae using limits.

References:

1) Differential Equations: With Applications and Historical Notes by G. F. Simmons

2) ROBERT G. BARTLE. DONALD R. SHERBERT. INTRODUCTION TO. REAL. ANALYSIS. THIRD EDITION.

Additional References

1) Ordinary Differential Equations (Dover Books on Mathematics) by M. Tenenbaum

2) Differential equation by G. F. Simmons and S. G. Krantz

3) Methods of Real Analysis by Goldberg

Semester II	**Course II**	**Course name: Algebra I**	**Course code 18US2MT2**

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Module 1	**Congruences** (11 lectures teaching load and 15 lectures students working load) (Maximum marks 15)
1.1	Congruence: Definition and elementary properties, simple examples using algebraic properties.
1.2	Euler phi-function (Totient) and examples. Euler's theorem (only statement). Fermat's little Theorem. Solving simple problems using these theorems. Wilson's theorem (only statement). Simple problems.
1.3	Introduction to $\mathbb{Z}_n$, addition and multiplication in $\mathbb{Z}_n$, multiplicative inverse in $\mathbb{Z}_n$. Decimal representation of an integer, Divisibility test for 3, 9 and 11, finding last digit.
Module 2	**Polynomials** (11 lectures teaching load and 15 lectures students working load) (Maximum marks 15)
2.1	Polynomials in one variable with real coefficients. Degree, leading coefficient and monic polynomial. Division Algorithm (without proof). G.C.D of two polynomials (Euclidean method)
2.2	Root and factor of a polynomial,

	multiplicity of a root,
	Remainder Theorem,
	Factor Theorem.
	Rational root theorem. Factorization over Q
2.3	Irreducible polynomial, Eisenstein's criteria (without proof).
	Number of real roots of nth degree polynomial is at most n.
	Reciprocal polynomial,
	Repeated root of a polynomial is also a root of its derivative.
	Relation between the roots and the coefficients of a polynomial. Examples.
Module 3	**COMPLEX NUMBERS(11 Lectures)**
	(11 lectures teaching load and 15 lectures students working load) (Maximum marks 15)
3.1	Review of a complex number,
	Polar representation.
	Argand diagram.
	Conjugate and its properties.
3.2	Fundamental theorem of algebra (only statement).
	Complex roots of a Real Polynomial occur in conjugate pairs.
	Factorization of a real polynomial as a product of linear and quadratic polynomials over $\mathbb{R}$. Odd degree polynomial has a real root.
	De-Moivre's Theorem.
	Roots of unity.
	Roots of a complex number.

Module 4	**Group theory** (12 lectures teaching load and 15 lectures students working load) (Maximum marks 15)
4.1	Group; definition and simple examples like $\mathbb{Q}$, $\mathbb{R}$ under addition, the group of n, nth roots of unity. The group Z_n under addition. Verification of $\mathbb{Z}_n^*$ being a group under multiplication. Abelian and non-abelian groups. Order of a group, order of elements of a group.
4.2	Subgroup, N& S. condition for a finite subset to be a subgroup (without proof), Testing whether $xHx^{-1} = H$ for a given x in the group G and H < G.
4.3	Permutation group: Permutations on n symbols. The group S_n under composition and $o(S_n) = n!$. Cycles and transpositions, representations of a permutation as a product of disjoint cycles, and as product of transposition (Without Proof). Listing permutations in the group S_3, S_4 etc. Sign of a permutation, odd and even permutations (Without Proof). A_n, the alternating subgroup of S_n. The group D_n of symmetry of a regular polygon for n=3 & 4.
4.4	Solving equations in a group. Partition of a positive integer, its relation to decomposition of a permutation as product of disjoint cycles, Conjugate of a permutation.

Projects

Proving Euler's theorem and Wilsons's theorem

Relationship between the Euler's totient function and o($\mathbb{Z}_n^*$, .) for n being a prime.

Chinese reminder theorem

Proof of Eisenstein's criterion for irreducible polynomials over $\mathbb{Z}$.

Complex plane.

Proving the identity element in a group is unique and inverse of an element is unique.

If every element of a group is of order less than 3 then the group is Abelian.

Group of even order contains an element of order 2 and there is odd number of such elements.

Expressing symmetries of a isosceles triangle, rectangle, regular polygon of n sides as a subgroup of S_n.

Refrences:

1) Elementary Number theory by David Burton Seventh Edition, McGraw Hill Education (India) Pvt Ltd.

2) Discrete Mathematics by Norman L. Biggs, Clarendon Press.

Additional References:

1) Introduction to theory of numbers by Niven and S. Zuckerman, Wiley Eastern.

2) A survey of Modern Algebra by G. Birkoff and S. Maclane Mac Milan

3) University Algebra by N. S. Gopalkrishnan, New Age International Ltd.

18US2MTP

Semester II

Mathematics Practical Paper

Calculus and Algebra

1. **Graphs of real valued functions of one variable**

2. **Limits and continuity of real valued functions of one variable**

3. **Differentiability of real valued functions of one variable**

4. **Applications of derivatives**

5. **Congruences**

6. **Polynomials**

7. **COMPLEX NUMBERS**

8. **Group theory**

Guide lines about conduct of Projects/Case Study.

1. **Projects/ Case Study/ Book Review :**
   Conduct and Evaluation: A student can submit a project/ Case Study/ do a Book Review in lieu of mid-semester test in a semester. The project should be 10 page typed pages in an A4 size paper with font size of 12. The topic of project should be selected in consultation of the teacher. **Maximum marks allotted for this is 30 and the remaining 10 marks are from Tutorial related activities.**

   The topic can be of expository / historical survey / interdisciplinary nature and the material covered in the project / case study should go beyond the scope of the syllabus. The student must clearly mention the sources (Book / on-line) used for the project/ case study. The use of Mathematical software should be encouraged. The project should be done under the supervision of a faculty in a college/ Institution / University.

   The following Marking scheme is suggested for evaluation of projects:

   30% marks for exposition
   20% marks for literature
   20% marks for Scope
   10% marks for originality
   20% marks for presentation.

   Continuous evaluation:
   Internal evaluation (40%):

   - There will be 40 marks continuous evaluation during Practical. A student can opt for projects or the student can do a book review, this will be evaluated out of 40 marks.
   - The project / book review will be under the guidance of the mentor allotted to the students by the head of the department.

   - There will be regular tests which can be of the form quiz/ descriptive test/ objective test/ group discussion etc.
   - Each test will be marked out of 10 marks.
   - The total score obtained in all of the above will finally be averaged to 40 marks. The 6 best score out of all the test will be considered while calculating the average score. (Subject to change.)

- A student should secure at least 40% marks to be eligible to get a passing grade in the Internal evaluation (out of 40 marks a student is required to get minimum of 16 marks).

Semester end Examination (60%):

At the end of the semester there will be a semester end exam carrying a maximum of 60 marks.
There will be 4 Questions one from each Unit. Each question will carry 15 marks (with option maximum of 25 marks). The question paper will cover the whole syllabus in such a way that a student will need to have understood each topic well to have secured 90% and above and an average student can at least secure a passing grade.

- A student should secure at least 40% marks to be eligible to get a passing grade (The student needs to secure minimum of 24 marks out of 60 to pass the course).

- A student who has failed to secure a passing grade /absent for any reason in the internal evaluation will have to give test out of 40 marks, consisting of Questions based on all the 4 units.
- ATKT internal and Semester Exam will have the same paper pattern as the regular exam. (Subject to change.)